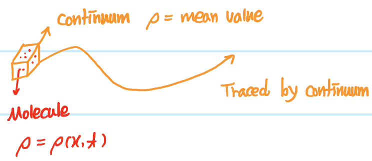
Continuum

Tuesday, August 22, 2017 13:50

For the glory of God



Continuum

- Air is made of discrete molecules which have own velocities.

  e.g. H_2, O_2, N_2 ; 6.023×10^{23} molecules in 1 mole

- In fluid dynamics, we are expected to follow flow trajectories for some reasons.

  Here, what if we have unsteady flow? Do we need to deal with each molecule w.r.t. changes in velocity?

- For this reason, we deal with average properties of a collection of particles rather than individual properties.

  This is called as Continuum. (Macroscopic quantity)

- In general, Continuum concept is only valid if Knudsen number is much less than 1.

  $$\text{Knudsen number} = \frac{\lambda}{c} \; ; \text{ where } \begin{cases} c : \text{Characteristic length} \\ \lambda : \text{Mean free path} \end{cases}$$

- Then, what is mean free path?

  - It is averaged distance traveled by a gas molecule between collisions with other particles.

    e.g. •⟷• ; λ is smaller than the one as below

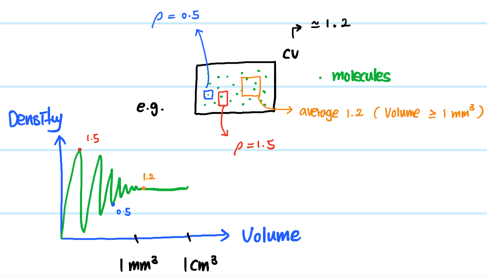
    • ⟵⟶ •

- $\frac{\lambda}{c} \lll 1$; Continuum is okay

  ↓ In other words

  $\lambda \lll c$: Continuum

  $\lambda \ggg c$: Rarefied gas



- In terms of Knudsen number,

  Continuum | Transitional | Free molecule → Knudsen number (0.2~0.3, 1)

  (Slip effect) : It is known that, for gaseous flow, there always exists a non-zero velocity near the wall.

  ↳ Velocity is no longer to be zero at the surface. From kinetic theory, $u_{wall} = L_{slip} \left(\frac{\partial u}{\partial n}\right)_{wall}$

  (L_{slip} ↓ Slip length)

Incompressible flow vs. Constant density flow

- In order to clarify the difference between incompressible and constant density flow, let us take old qual questions.

  - Incompressible flow means density is constant. Do you agree or not? ⇒ Disagree

  - If the flow is assumed to be incompressible flow, can the density change? ⇒ Yes it can, if other effects present

- Basically, incompressible flow means :

  When P is exerted on the flow, there is no change in density. $\therefore \frac{d\rho}{dP} = 0 \Leftrightarrow \rho \neq f(P)$

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The density can be changed but neglected because the rate of change is too small.

(e.g. Generally speaking, changes less than 5% in density $\Leftrightarrow \rho = const$)

Let's say we have water and oil. (Both incompressible fluid)

→ If we mix them together to make an emulsion.

And then, we pour them out of a cup.

In this case, the flow is incompressible but the density is not constant because there are density gradients

· Therefore,

Incompressible flow $\underset{\text{Always}}{\overset{\text{Not always}}{\rightleftarrows}}$ Constant density flow

In other words, from the fundamental expression of speed of sound in a gas,

$\uparrow a^2 = \left.\frac{dP}{\downarrow d\rho}\right|_s$ $\quad\therefore$ when low Mach number, $M \downarrow = \frac{V}{a \uparrow}$

↳ less change in density

Q. Why is the air less dense higher up you go?

· On ground, there are much more molecules hovering.

· More momentum exchange, high temperature, so high pressure, it ends up increasing density. ($p = \rho R T$)

· $\rho = f(p, T)$ but $\frac{d\rho}{dp}$ is greater and $\frac{d\rho}{dT}$ might be negligible.

* Atmosphere model by ICAO standard

$T = 288.15 - 0.0065h$ [K]

$p = 101.325 \left(\frac{T}{288}\right)^{5.3559}$ [kPa]

when $h \le 11$ km