# Equations of motion

Friday, November 3, 2017 01:44

*For the glory of God*

## Assumptions

- An aircraft is represented as a moving point mass.
- There is no side slip.

Geometry is collapsed to a point. 

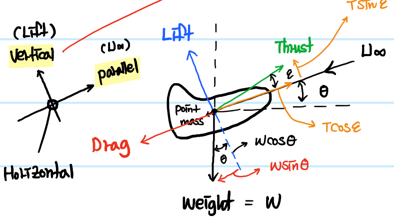
In a point mass analysis, we only think about the output.

(e.g. Do not care about area but care about $C_{D,0}$)

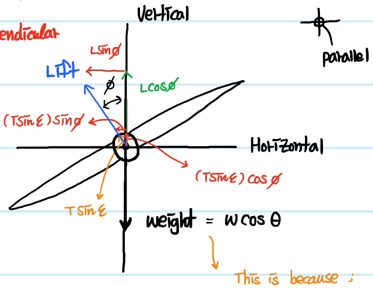
- Therefore, we don't care about both Aerodynamics (e.g. Aerodynamic center) and Stability (e.g. pitch angle effect)

## Free body diagram

This is why lift is defined as a force perpendicular to $U_\infty$




< Side view >




< Front view >

This is because ; vertical line of front view = vertical line of side view

; where

ø = bank angle

θ = flight path angle

ε = Thrust vector angle

## Equations of motion

- Let's apply Newton's 2<sup></sup>ND law to each direction : parallel, vertical, and horizontal

↳ when you think about it, please come up with Right hand rule.

The force is the time rate of momentum change ;

$$\vec{F} = \frac{d}{dt}(m\vec{U})$$

$$= \frac{dm}{dt}\vec{U} + m\frac{d\vec{U}}{dt}$$

$\frac{dm}{dt} \to 0$ (∵ we are now looking into the snapshot of the flying)

- ① When it comes to Parallel direction, (please see the side view) → ⊕

$$\vec{F} = m\frac{d\vec{U}}{dt}$$

$$\Leftrightarrow T\cos\varepsilon - D - W\sin\theta = m\frac{dU_\infty}{dt}$$

; where $U_\infty$ = General expression at this point

- ② Regarding vertical direction, (please see the front view) ↑ ⊕

$$\vec{F} = m\frac{d\vec{U}}{dt}$$

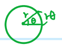
$$\Leftrightarrow L\cos\phi - W\cos\theta + T\sin\varepsilon\cos\phi = m\frac{U_\infty^2}{R}$$

$\Leftrightarrow L\cos\phi - W\cos\theta + T\sin\varepsilon\cos\phi = m\frac{[illegible]}{R}$

↓

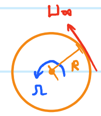
cf. Angular velocity ($\omega$)

This is based on ;



$\omega \equiv \frac{d\theta}{dt}$ ; $\omega = \Omega$ (interchangeable)

$V \equiv \frac{s}{t} = \frac{r\Delta\theta}{\Delta t} = \omega r \Leftrightarrow \omega = \Omega = \frac{V}{r}$

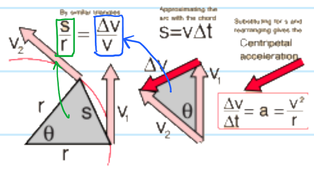



$U_\infty \equiv \Omega R \Leftrightarrow \Omega = \frac{U_\infty}{R}$

$a \equiv \Omega^2 R$

$= \frac{U_\infty^2}{R^2} R$

$= \frac{U_\infty^2}{R}$



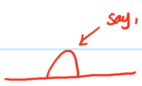

· In terms of ③ horizontal direction,

$\vec{F} = m\frac{d\vec{U}}{dt}$

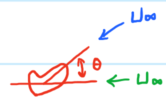
$\Leftrightarrow m\frac{(U_\infty\cos\theta)^2}{R} = L\sin\phi + T\sin\varepsilon\sin\phi$

↳ Note that ⊕ ← ⊖ — horizontal line

This is because ;




vs.




; $\theta$ = flight path angle

$U_\infty$ ↳ velocity to the flight path

$U_\infty$ ↳ velocity to the horizontal line

## Velocity and Airspeed

Which way is $V_\infty$ going?

In situations where we are interested in developing equations of motion we will show the velocity (airspeed), $V_\infty$, in the direction of motion:


In situations where we are interested in aerodynamic forces we will show the airspeed (velocity), $V_\infty$, into the body:


In this course we will use velocity and airspeed interchangeably. Be aware you may not be able to do this in situations with wind.