

# Particle swarm optimization

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For the glory of God

## Introduction

- In 1980, people tried to mimic swarming in Biology and optimization wasn't considered at that time.

↳ They thought each of individual would follow the Simple Rule when they observed birds or fishes swarming.

e.g. Rule 1 (Separation) : Keep the distance from its neighbors

- Kennedy and Eberhart thought that it would be possible to use the rules as optimization.

↓

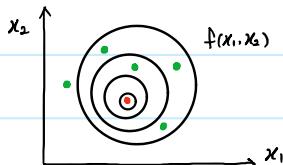
- They ended up introducing the "Rooster" concept.

The rule can be applied to UAVs flight

↳ It is defined as a desired location that all swarming particles want to go.

## Particle Swarm Optimization (PSO) : The basic idea

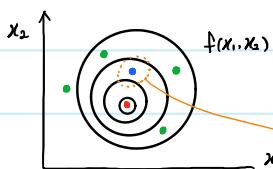
- Step 1 : Create a population of particles over the design space



- : particles
- : optimum

Here, we assume that  $V_{initial} = 0$

- Step 2 : Evaluate each particle and determine the best particle



- : particles
- : optimum
- : Best particle at step 2

↳ The particle could be our first Rooster!

⇒ So, every particle would have a direction toward the Rooster.

(Algorithm says that all particles eventually find the Rooster)

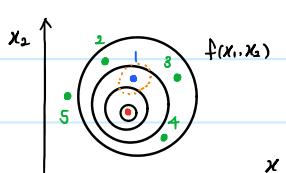
- Step 3 : Calculate new velocities

- New velocities can be calculated by the equation :

$$V_i^{t+1} = V_i^t + \varphi_1 U_1^t (p_{bi}^t - x_i^t) + \varphi_2 U_2^t (g_b^t - x_i^t) ; \text{ where } \begin{cases} p_{bi} : \text{Personal best} \\ g_b : \text{Global best} \end{cases}$$

where  $i = \text{particle}$   
 $V_i^{t+1}$  is new velocity with time increment  
 $V_i^t$  is old velocity  
 $\varphi$  is trust (or weight factor) → Determine either following the personal guidance or the global guidance.  
 $U$  is a sort of sample from uniform distribution [0, 1] → stochastic term

At this point, we have five particles :



- : particles
- : optimum
- : Best particle at step 2

Weight factors may be different to weight more

something than others.

1)  $V_i^{t+1} = V_i^t + \varphi_1 U_1^t (p_{bi}^t - x_i^t) + \varphi_2 U_2^t (g_b^t - x_i^t)$

$$= 0 + (\varphi_1 \text{U}_1' (x_1' - x_1) + \varphi_2 \text{U}_2' (x_2' - x_2))$$

$$= 0 + 0 + 0 \quad \because \text{particle 1 doesn't go anywhere because } V_1^2 = 0$$

↓  
At particle 1, it is the personal best itself at this point.

We assume that  $V_{int} = 0$       particle 1 was our the best particle (= Global best)

$$2) V_2^2 = V_2' + \varphi_1 \text{U}_1' (p_{b1}' - x_2') + \varphi_2 \text{U}_2' (g_b' - x_2')$$

$$= 0 + (\varphi_1 \text{U}_1' (x_1' - x_2) + \varphi_2 \text{U}_2' (x_1' - x_2))$$

$$= 0 + 0 + \varphi_2 \text{U}_2' (x_1' - x_2) \quad \because \text{Finally, particle 2 has a velocity}$$

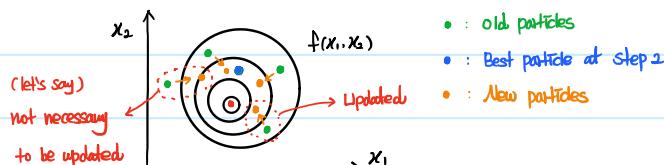
↓  
We assume that  $V_{int} = 0$       For initial step, they ( $1 \sim 5$  particles) ate all personal best

3), 4), 5) will have the same result with 2) but different i notation.

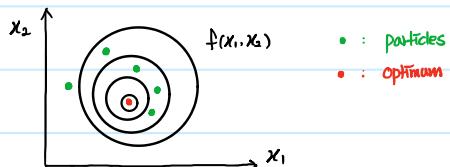
Step 4 : move to new position as necessary

- Based on :      ↳ If a new point is better than the old point, update it.

$$x_i^{t+1} = x_i^t + v_i^{t+1}$$



↓  
Hence, we have



Step 5 : Go to step 2 and repeat until stopping criteria satisfied

Note that :

- In SPO, we need to use a penalty function for constraints because there is no way to incorporate them.
- There is a lot of modifications of SPO.
- SPO would be good for multiple minimum problems.