Transportation Problem (TP)

Wednesday, May 23, 2018 10:25 PM

For the glory of God

Transportation Problem - Case study

a) Introduction

- Procter & Gamble company made over 300 brands of consumer goods worldwide.
- As the company moved toward global brands, they realized that it needed to consolidate plans to reduce costs.

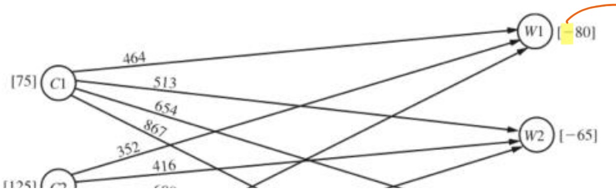
  ↳ The study focused on redesigning the company's production and distribution system for its operations.
- They finally saved over $200 million in pretax costs per year.
- How was it possible?

  → They actually solved Transportation problems.

b) Problem description

- One of the main products of the company is canned peas.
- The peas are prepared at three canneries and then shipped by truck to four distributing warehouses.
- Let's say:
  - An estimate has been made of the output from each cannery.
  - Each warehouse has been allocated a certain amount.
- The problem is now to determine which plan for assigning these shipments to the various combinations minimizing total cost.

- By ignoring the geographical layout, we can provide a network representation of this problem in a simple way by lining up all the canneries in one column on the left and all the warehouses in one column on the right.



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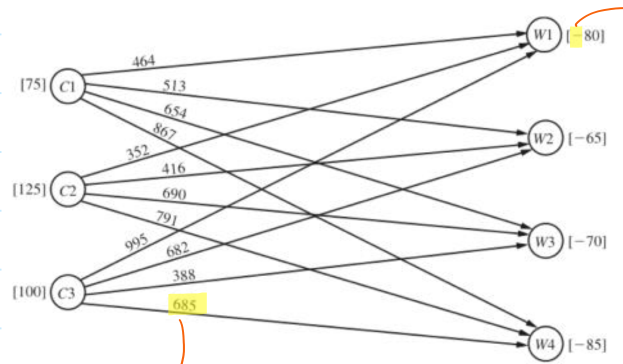


· Shipping data will be given as below ;

■ TABLE 8.2 Shipping data for P & T Co.

		Shipping Cost ($) per Truckload				
		Warehouse				
		1	2	3	4	Output
Cannery	1	464	513	654	867	75
	2	352	416	690	791	125
	3	995	682	388	685	100
Allocation		80	65	70	85	

if 1→1 route

→ Total 300 products Truckload

↳ total 300 allocation

c) Mathematical model

Minimize $Z = 464x_{11} + 513x_{12} + 654x_{13} + 867x_{14} + 352x_{21} + 416x_{22} + 690x_{23} + 791x_{24} + 995x_{31} + 682x_{32} + 388x_{33} + 685x_{34},$

subject to the constraints

$$
\left.\begin{aligned}
x_{11} + x_{12} + x_{13} + x_{14} &= 75\\
x_{21} + x_{22} + x_{23} + x_{24} &= 125\\
x_{31} + x_{32} + x_{33} + x_{34} &= 100
\end{aligned}\right\}\ \text{Ship out from company}
$$

$$
\left.\begin{aligned}
x_{11} + x_{21} + x_{31} &= 80\\
x_{12} + x_{22} + x_{32} &= 65\\
x_{13} + x_{23} + x_{33} &= 70\\
x_{14} + x_{24} + x_{34} &= 85
\end{aligned}\right\}\ \text{Ship into warehouse}
$$

and

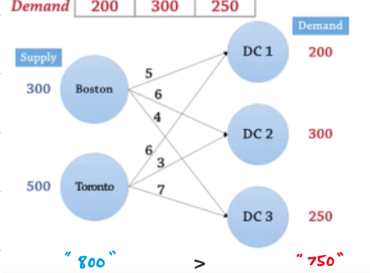
$x_{ij} \geq 0 \quad (i = 1, 2, 3; j = 1, 2, 3, 4).$

· In this case, there is a balance between the total supply from all sources and the total demand at all destinations.

↳ such that $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$; this is called as the balanced transportation problem.

· However, in some real problems, it might be unbalanced transportation problem.

From \ To	DC1	DC2	DC3	Supply
Boston	5	6	4	300
Toronto	6	3	7	500
Demand	200	300	250	



"800" > "750"

X_{ij} = # units shipped from Plant i to DC j

i = B(Boston), T(Toronto) j = 1(DC1), 2(DC2), 3(DC3)

Min $5X_{B1} + 6X_{B2} + 4X_{B3} + 6X_{T1} + 3X_{T2} + 7X_{T3}$

Subject to:

Due to unbalance ..

$X_{B1} + X_{B2} + X_{B3} \leq 300$ *(Boston's Supply)*

$X_{T1} + X_{T2} + X_{T3} \leq 500$ *(Toronto's Supply)*

$X_{B1} + X_{T1} = 200$ *(DC1's Demand)*

$X_{B2} + X_{T2} = 300$ *(DC2's Demand)*

$X_{B3} + X_{T3} = 250$ *(DC3's Demand)*

$X_{B1}, X_{B2}, X_{B3}, X_{T1}, X_{T2}, X_{T3} \geq 0$ *or* $X_{ij} \geq 0$

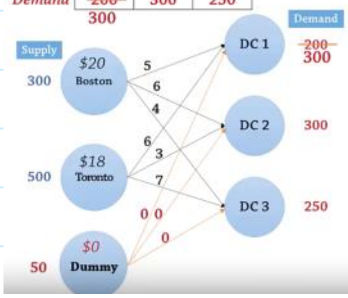
· For unbalanced problem, we need to reformulate the problem by introducing dummy (or slack) variables.

· Let's say that DC1 demand changed from 200 to 300 so that $\Sigma s_i < \Sigma d_i$

From \ To	DC1	DC2	DC3	*Supply*
Boston	5	6	4	300
Toronto	6	3	7	500
Demand	~~200~~ 300	300	250	

X_{ij} = # units shipped from Plant i to DC j

$i = B$(Boston), T(Toronto), D(Dummy) $\quad j = 1$(DC1), 2(DC2), 3(DC3)



Min $25X_{B1} + 26X_{B2} + 24X_{B3} + 24X_{T1} + 21X_{T2} + 25X_{T3} + 0X_{D1} + 0X_{D2} + 0X_{D3}$

Subject to:

$X_{B1} + X_{B2} + X_{B3} \le 300$ *(Boston's Supply)*

$X_{T1} + X_{T2} + X_{T3} \le 500$ *(Toronto's Supply)*

$X_{D1} + X_{D2} + X_{D3} \le 50$ *(Dummy's Supply)*

$X_{D1} + X_{B1} + X_{T1} = \cancel{200}\,300$ *(DC1's Demand)*

$X_{D3} + X_{B2} + X_{T2} = 300$ *(DC2's Demand)*

$X_{D3} + X_{B3} + X_{T3} = 250$ *(DC3's Demand)*

$X_{ij} \ge 0$

d) Linear programming - Simplex method

· Transportation problem is indeed a special type of linear programming problem.

· Let's take an example. (The book given from 정창윤 박사)

Example 5.1-1

MG Auto has three plants in Los Angeles, Detroit, and New Orleans, and two major distribution centers in Denver and Miami. The capacities of the three plants during the next quarter are 1000, 1500, and 1200 cars. The quarterly demands at the two distribution centers are 2300 and 1400 cars. The mileage chart between the plants and the distribution centers is given in Table 5.1.

The trucking company in charge of transporting the cars charges 8 cents per mile per car. The transportation costs per car on the different routes, rounded to the closest dollar, are given in Table 5.2.

The LP model of the problem is given as

$$\text{Minimize } z = 80x_{11} + 215x_{12} + 100x_{21} + 108x_{22} + 102x_{31} + 68x_{32}$$

TABLE 5.2 Transportation Cost per Car

	Denver (1)	Miami (2)
Los Angeles (1)	$80	$215
Detroit (2)	$100	$108
New Orleans (3)	$102	$68

subject to

unimodular property

$$\begin{aligned}
x_{11} + x_{12} \quad &= 1000 \quad \text{(Los Angeles)}\\
x_{21} + x_{22} \quad &= 1500 \quad \text{(Detroit)}\\
+ x_{31} + x_{32} &= 1200 \quad \text{(New Orleans)}\\
x_{11} + x_{21} + x_{31} &= 2300 \quad \text{(Denver)}\\
x_{12} + x_{22} + x_{32} &= 1400 \quad \text{(Miami)}\\
x_{ij} \ge 0,\ i = 1, 2, 3,\ j &= 1, 2
\end{aligned}$$

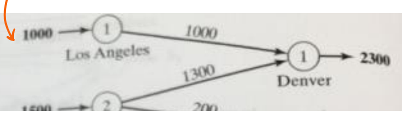
These constraints are all equations because the total supply from the three sources (= 1000 + 1500 + 1200 = 3700 cars) equals the total demand at the two destinations (= 2300 + 1400 = 3700 cars).

The LP model can be solved by the simplex method. However, with the special structure of the constraints we can solve the problem more conveniently using the **transportation tableau** shown in Table 5.3.

TABLE 5.3 MG Transportation Model

	Denver	Miami	Supply
Los Angeles	80 x_{11}	215 x_{12}	**1000**
Detroit	100 x_{21}	108 x_{22}	**1500**
New Orleans	102 x_{31}	68 x_{32}	**1200**
Demand	**2300**	**1400**	

Linear programming was performed



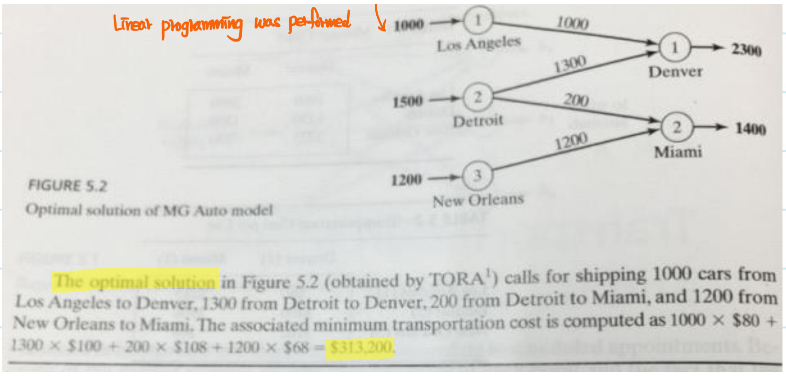


FIGURE 5.2
Optimal solution of MG Auto model

The optimal solution in Figure 5.2 (obtained by TORA[1]) calls for shipping 1000 cars from Los Angeles to Denver, 1300 from Detroit to Denver, 200 from Detroit to Miami, and 1200 from New Orleans to Miami. The associated minimum transportation cost is computed as 1000 × \$80 + 1300 × \$100 + 200 × \$108 + 1200 × \$68 = \$313,200.

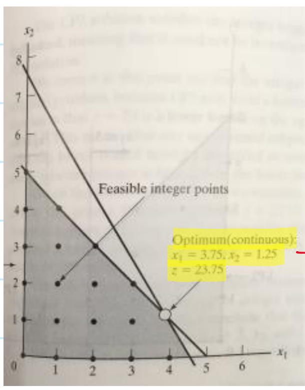
e) Integer linear programming

· Let's say that we need to solve ;

- maximize $z = 5x_1 + 4x_2$

- Subject to $\begin{cases} x_1 + x_2 \le 5 \\ 10x_1 + 6x_2 \le 45 \end{cases}$

· We will obtain the solution as following ;



→ Let's say ; we have to obtain integer variables with respect to x_1 and x_2.

· How would you be able to solve the problem using linear programming ?

⇒ Branch-and-Bound (B&B) algorithm has been widely used in Industrial Engineering.

· The basic idea is to break the range into integer type. For example, Since the optimum was $x = 3.75$, let's break into ;

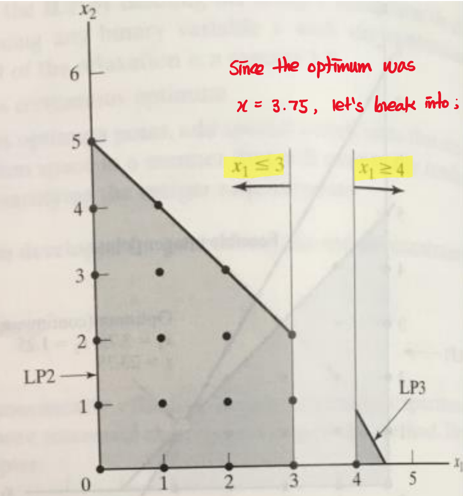


FIGURE 9.7
Solution Spaces of LP2 and LP3 for Example 9.2-1

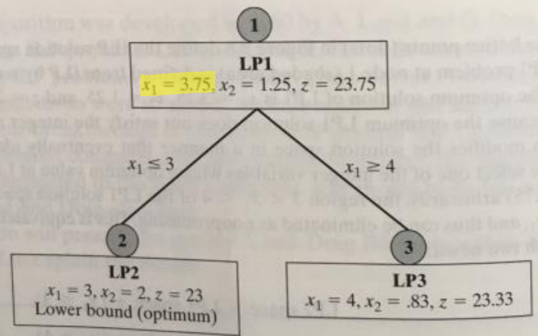


FIGURE 9.8
Using branching variable x_1 to create LP2 and LP3 for Example 9.2-1

subject to

$$
\begin{aligned}
x_1 + \quad x_2 &\leq 5 \\
10x_1 + 6x_2 &\leq 45 \\
x_1 \qquad &\leq 3 \\
x_1, x_2 &\geq 0
\end{aligned}
$$

· Repeat to generate the tree until an optimum is achieved.

Chapter 9 Integer Linear Programming

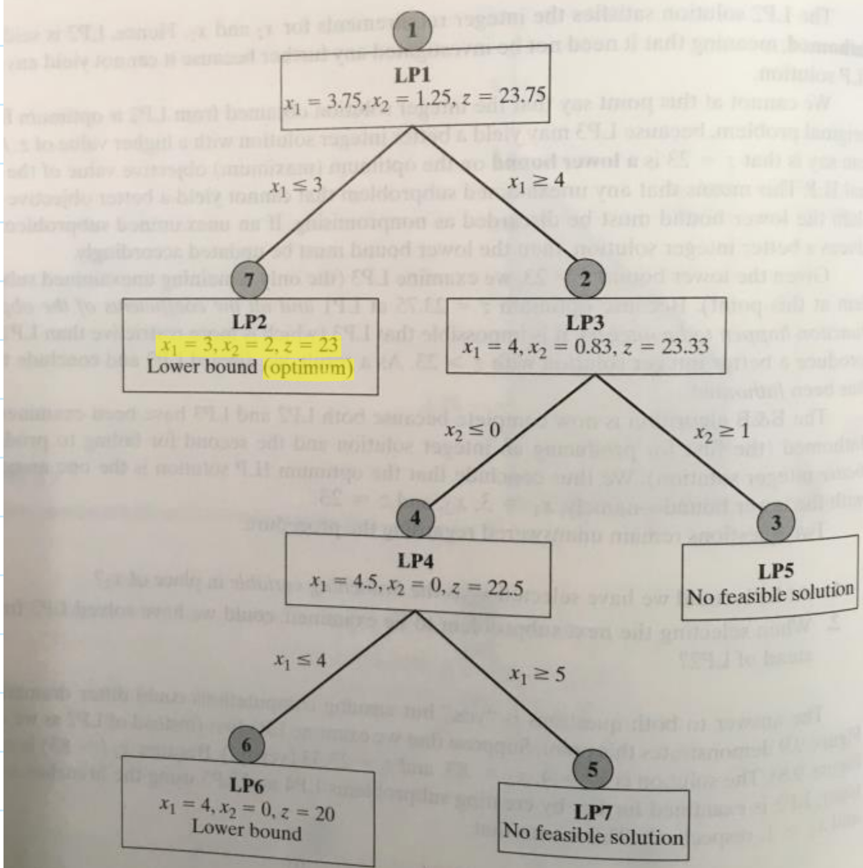


FIGURE 9.9
Alternative B&B tree for Example 9.2-1

· Fortunately, Transportation Problem doesn't require integer linear programming because it has been proven that it never happen to have float point.

f) Big - M method

· In operation research, the Big M method is a method of solving linear programming problems using the simplex method.

· The method starts with the Linear Programming form { · All the constraints are equations with non-negative righthand side.
· All the variables are non-negative.

· Then, when does the method need to be used?

e.g. maximize $z = 3x_1 + 5x_2$

subject to $x_1 \le 4 \rightarrow x_1 + x_3 = 4$

$2x_2 \le 12 \rightarrow 2x_2 + x_4 = 12$

$3x_1 + 2x_2 = 18$

$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ x_4 \ge 0$

(slack variables)

$\Rightarrow$

x_1	x_2	x_3 ↓	x_4 ↓	b
1	0	1	0	4
0	2	0	1	12
3	2	0	0	18
3	5	0	0	z

· Unfortunately, these equations do not have an obvious initial solution because there is no longer a slack variable to use as the initial basic variable.

↳ However, it is well known that it is necessary to find an initial solution to start the simplex method.

to use as the initial basic variable.

↳ However, it is well known that it is necessary to find an initial solution to start the simplex method.

· This difficulty can be circumvented by Big M method.

↳ An artificial variable is added to form a starting solution similar to all-slack basic solution.

· However, because the artificial variables aren't part of the original LP model,

- They are assigned a very high penalty in the objective function.
- Thus, forcing them (Eventually) to equal zero in the optimum solution.
- This will always be the case if the problem has a feasible solution.

1. Apply the **artificial-variable technique** by introducing a *nonnegative* **artificial variable** (call it $\bar{x}_5$)[14] into Eq. (3), just as if it were a slack variable

(3) $\quad 3x_1 + 2x_2 + \bar{x}_5 = 18.$

2. Assign an *overwhelming penalty* to having $\bar{x}_5 > 0$ by changing the objective function
$Z = 3x_1 + 5x_2$ to

$Z = 3x_1 + 5x_2 - M\bar{x}_5,$

{ $-M$: maximization problem
{ M : minimization problem

where M symbolically represents a *huge* positive number. (This method of forcing $\bar{x}_5$ to be $\bar{x}_5 = 0$ in the optimal solution is called the **Big *M* method.**)

Now find the optimal solution for the real problem by applying the simplex method to the artificial problem, starting with the following initial BF solution:

Initial BF Solution
Nonbasic variables: $\quad x_1 = 0, \quad x_2 = 0$
Basic variables: $\quad x_3 = 4, \quad x_4 = 12, \quad \bar{x}_5 = 18.$

Because $\bar{x}_5$ plays the role of the slack variable for the third constraint in the artificial problem, this constraint is equivalent to $3x_1 + 2x_2 \le 18$ (just as for the original Wyndor Glass Co. problem in Sec. 3.1). We show below the resulting artificial problem (before augmenting) next to the real problem.

The Real Problem

Maximize $\quad Z = 3x_1 + 5x_2,$
subject to
$$x_1 \le 4$$
$$2x_2 \le 12$$
$$3x_1 + 2x_2 = 18$$
and
$$x_1 \ge 0, \quad x_2 \ge 0.$$

The Artificial Problem

Define $\quad \bar{x}_5 = 18 - 3x_1 - 2x_2.$
Maximize $\quad Z = 3x_1 + 5x_2 - M\bar{x}_5,$
subject to
$$x_1 \le 4$$
$$2x_2 \le 12$$
$$3x_1 + 2x_2 \le 18$$
$$(\text{so} \quad 3x_1 + 2x_2 + \bar{x}_5 = 18)$$
and
$$x_1 \ge 0, \quad x_2 \ge 0, \quad \bar{x}_5 \ge 0.$$

· Let's apply the Big M method to a real problem given from CJ Logistics.

Suwon sub [0~9] ⟶ Yongin hub [1, 2]

Incheon sub [0~9] $\quad$ Daejeon hub [0, 9]

Seoul sub [0~9] $\quad$ ⋮

⋮

Even though Suwon sub has [0~9] classification, Yongin hub only accepts [1] and [2].

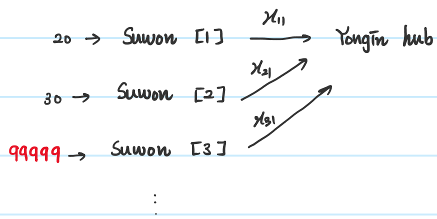
↳ It is challenging to consider every situation, e.g., Suwon [3] never goes to Yongin hub in Transportation problem.

∴ For transportation problem, we just force Suwon [3] to go Yongin hub but use Big M method such that ;

20 → Suwon [1] $\xrightarrow{x_{11}}$ Yongin hub

x_{21}

· Also, in Transportation Problem, Every node should connect to each other.



· Also, in Transportation Problem, Every node should connect to each other.

· I may be able to delete the connection (e.g. Suwon [3] to Yongin) when determining design variables ; however, it will be trouble when the size of problem becomes bigger.

objective function : $20x_{11} + 30x_{21} + 99999x_{31} + \cdots$

↳ Since we're doing minimization, there must be little case to go from Suwon [3] to Yongin hub.

9) Case study - CJ Logistics

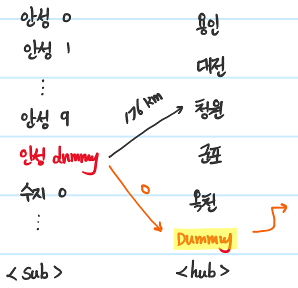
· All data (May 17, 2018) were given from CJ Logistics in order to validate my code is working well.

· After summarizing all the data, I realized that ;

- Supply is larger than demand (In my case, this is hub capacity).

· Thus, I introduced the dummy hub to obtain an optimal solution.

(without the dummy, I got "Infeasible solution", though)



cf. Supply ≃ 700,000

Hub Capacity ≃ 620,000

Remains are expected to arrive at this dummy.

· Hence, I would get the mathematical expression ;

coefficient for the dummy

objective function $= x_1 \cdot 99999 + x_2 \cdot 176 + \cdots + x_{n-1} \cdot 0 + \cdots$

coefficient for Big-M method

Subject to.

Supply : $x_1 + x_2 + \cdots + x_{n-1} + \cdots = $ Sub Supply

demand : $x_{11} + x_{12} + \cdots + x_{1,n-1} + \cdots = $ Sum of the input heading to dummy sub

Variable : $x_{ij} \geq 0$

· Finally, I could get the optimum.

↳ Just in case, I tested the case with 99999 coefficients instead of 0.

: Eventually, I had seen that the code gave multi-solutions.

e.g. 이천 → 대전 1597 물량 (99999 case)

이천 → Dummy 1597 물량 (0 case)

↳ Even though they had different routes, in the end, the objective function values were same.

1) $y = 0 \cdot x_1 + \cdots$ No need to consider x_1 value because it'd be canceled out

2) $y = 99999 x_1 + \cdots$ Make a summation but remove the value if 물량 goes to dummy with 99999 values.