

# Flow separation and Suction/Blowing

Monday, September 11, 2017 15:57

For the glory of God

## f) Flow Separation

- As we know, the flow separation occurs when the flow in boundary layer travels far enough against an adverse pressure gradient that the speed of the flow in B.L. relative to the object falls almost to zero.
- The fluid flow becomes detached from the surface of the object. (Vorticity is generated at the wall by viscosity  $\rightarrow$  It is transported to distance far from wall)
- Okay, this is wordy expression to explain flow separation.  
How would you define it mathematically?  $\oplus$
- The first thing we need to do is to define the shear stress at the wall. ( $u=0, v=0$ )
- Recall the shear stress tensor,

$$\sigma_{ij} = 2\mu (S_{ij} - \frac{1}{3} \Delta \delta_{ij}) \quad \begin{matrix} \oplus \\ \text{if we assume} \\ \text{the constant} \\ \text{density flow} \end{matrix}$$

$$\text{where } S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

If we think about the 2D flow,



$$\delta_{21} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

↑  
21 means y direction, x flow  
→  
0 ( $\because$  Impermeability)

$$\therefore \delta_{21} = Z_w = \mu \frac{\partial u}{\partial y}$$

or  $v=0$  for this case, only  $x$  flow

Then, what is the definition of the flow separation?

: It could be  $Z_w = \mu \frac{\partial u}{\partial y} \Big|_{y=0} = 0$

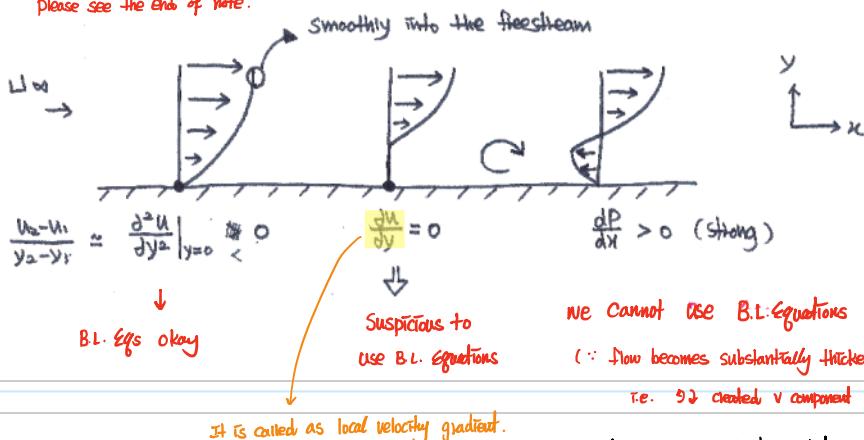
Let's think about the Boundary Layer Equation again.

$x$ -momentum :  $\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{dp}{dx} + \mu \frac{\partial^2 u}{\partial y^2}$

Since we consider the wall, LHS = 0 ( $\because u=0$  and  $v=0$ ) at  $y=0$

\* For more details,  
please see the end of note.

$$\frac{\partial^2 u}{\partial y^2} \Big|_{y=0} = \frac{1}{\mu} \frac{dp}{dx} ; \frac{\partial^2 u}{\partial y^2} = 0 : \text{Point of inflection}$$



For suction, is similarity solution possible?

: Yes but under some circumstances

## §. Flat plate with wall suction or blowing

The Blasius solution can be extended to non-zero wall

velocity,  $U_w \ll U_\infty$ , either positive (blowing) or negative (suction)

: For porous surface,

It is called as local velocity gradient.

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: Yes but under some circumstances

### §. Flat plate with wall suction or blowing

The Blasius solution can be extended to non-zero wall

velocity,  $v_w \ll U_\infty$ , either positive (blowing) or negative (suction)

: For porous surface,

$$v(x, y=0) = v_w \neq 0 \quad \begin{cases} \text{suction if } v_w < 0 \\ \text{blowing if } v_w > 0 \end{cases}$$

This could be applied to many application problems such as

boundary layer control, airfoil cooling, ablation, and so forth.

So, how could we handle this problem?

: We might be still able to use the concept / definition from Blasius.

$$\text{Let } \gamma \equiv y \sqrt{\frac{U_\infty}{2v_x}} \quad \text{and} \quad \eta \equiv \sqrt{2v_x x} f(\gamma)$$

Since it is defined as,

$$u = \frac{d\eta}{dy} \quad \text{and} \quad v = -\frac{df}{dx}$$

we ended up having those expressions as following;

$$u = U_\infty f'(\gamma) \quad v = \sqrt{\frac{U_\infty}{2x}} (\eta f''(\gamma) - f'(\gamma))$$

- Since we deal with the suction case now, the boundary condition should be changed comparing with Blasius one.

$$v = V_w \text{ at } y=0 \rightarrow V_w = \sqrt{\frac{U_{\infty} x}{2\lambda}} (0 - f(0))$$

(which means at  $\eta=0$ )

- Then, we have

$$f(0) = \frac{-V_w}{\sqrt{U_{\infty} x / 2\lambda}} \neq 0 \quad (\text{For Blasius, } f(0) = 0)$$

- You may wonder how the velocity profile looks like. Let's see!

: First of all, let's think about the case which is not the flat plate and allow  $dp/dx > 0$ , for instance;



From the B.L.  $x$ -momentum Equation, we have

$$U \frac{du}{dx} + V \frac{du}{dy} = - \frac{1}{\rho} \frac{dp}{dx} + V \frac{d^2 u}{dy^2}$$

At the wall,

$$U = 0 \text{ and } V = V_w \text{ at } y = 0$$

↳ please keep in mind that this is



- Then, the momentum equation becomes :

$$0 + V_w \frac{du}{dy} \Big|_{y=0} = - \frac{1}{\rho} \frac{dp}{dx} + V \frac{d^2 u}{dy^2} \Big|_{y=0}$$

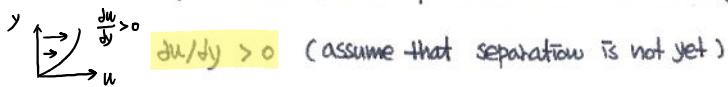
- By rearranging the equation,

$$\frac{d^2 u}{dy^2} \Big|_{y=0} = \frac{1}{V} \left( V_w \frac{du}{dy} \Big|_{y=0} + \frac{1}{\rho} \frac{dp}{dx} \right)$$

(Based on the condition)

; where  $V_w < 0$  (definition)

$\frac{dp}{dx} > 0$  (we setup this condition from the beginning)



- Since the suction tends to make  $\frac{d^2 u}{dy^2} \Big|_{y=0} < 0$  or at least (Based on the condition)

less positive, e.g. separation delay,  $\frac{du}{dy} \Big|_{y=0}$  should be

increasing as  $dp/dx$  increases.

- For instance, let's say ) this is why:

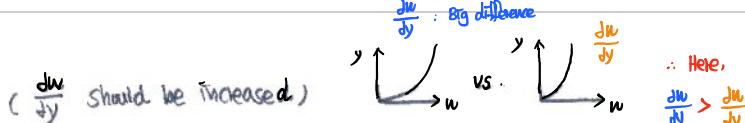
$V = \text{constant (positive)}, V_w = -10, \frac{dp}{dx} > 0, \rho = 1.225$

$$\frac{d^2 u}{dy^2} \Big|_{y=0} = \frac{1}{\text{constant}} \left( -10 \times \frac{du}{dy} \Big|_{y=0} + \frac{1}{1.225} \frac{dp}{dx} \right)$$

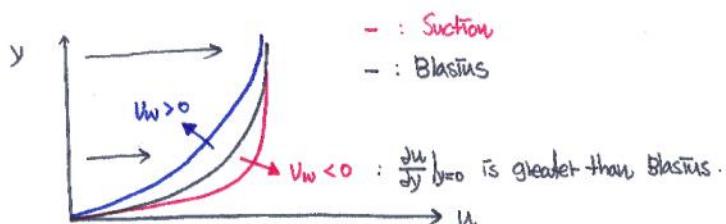
$\downarrow \qquad \downarrow$  will be increased

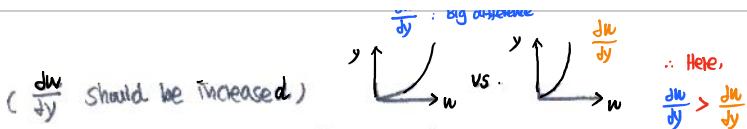
It should be increased

↳ In order to maintain  $\frac{du}{dy} > 0$

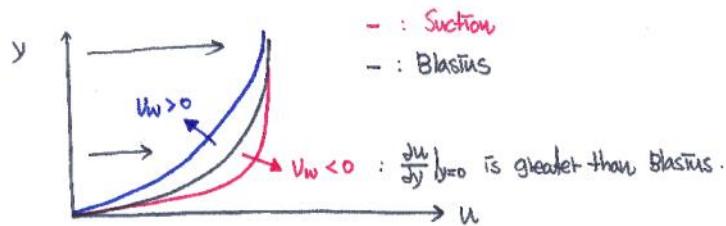


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- In other words,

Adverse pressure gradient makes the flow have less mixing.

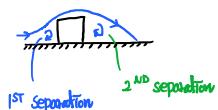
- ① Suction : what it does is to suck low momentum flow near the wall.  
↳ Then, high momentum flow at edge flows down.

- It thins the boundary layer and greatly increase the wall slope.
- It will be very stable and delay transition to turbulence (Remove slower-flowing fluid)

## ② Blowing

- It thickens the boundary layer.
- It will be less stable and prone to transition to turbulence.

of Urban engineering



## Flow separation (2017 AE6009)

- What is the effect of pressure gradient on the velocity profile?

- Let's derive mathematical expression as the first step.

### 1) Momentum equation

$$\rho \frac{D\vec{U}}{Dt} = \rho \vec{F} + \nabla \cdot \vec{\tau}_{IJ} \Leftrightarrow \rho \frac{DU}{Dt} = \rho F_r + \frac{dZ_{IJ}}{dx_F}$$

### 2) Stoke's contribution

He suggested :  $Z_{IJ} = \text{normal stress} + \text{shear stress}$

$$\begin{aligned}
&= -P \delta_{IJ} + \Delta_{IJ} \\
&= -P \delta_{IJ} + \mu \left( \frac{\partial u_I}{\partial x_J} + \frac{\partial u_J}{\partial x_I} \right) + \lambda (\nabla \cdot \vec{u}) \delta_{IJ} \\
&\approx -P \delta_{IJ} + \mu \left( \frac{\partial u_I}{\partial x_J} + \frac{\partial u_J}{\partial x_I} \right) + \left(-\frac{2}{3} \mu\right) (\nabla \cdot \vec{u}) \delta_{IJ} \\
&\approx -P \delta_{IJ} + \mu (\nabla \cdot \vec{u} + \nabla \cdot \vec{u}) - \frac{2}{3} \mu (\nabla \cdot \vec{u}) \delta_{IJ} \\
&\approx -P \delta_{IJ} + \mu \nabla \cdot \vec{u} + \frac{1}{3} \mu \nabla \cdot \vec{u} \delta_{IJ}
\end{aligned}$$

Hence, we have

$$\begin{aligned}
\rho \frac{D \vec{u}}{Dt} &= \rho \vec{F} + \nabla \cdot (-P \delta_{IJ} + \mu \nabla \cdot \vec{u} + \frac{1}{3} \mu \nabla \cdot \vec{u} \delta_{IJ}) \\
&= \rho \vec{F} - \nabla P + \mu \nabla^2 \vec{u} + \frac{2}{3} \mu \nabla \Delta \quad ; \text{ where } \Delta = \nabla \cdot \vec{u}
\end{aligned}$$

### 3) Boundary layer equation

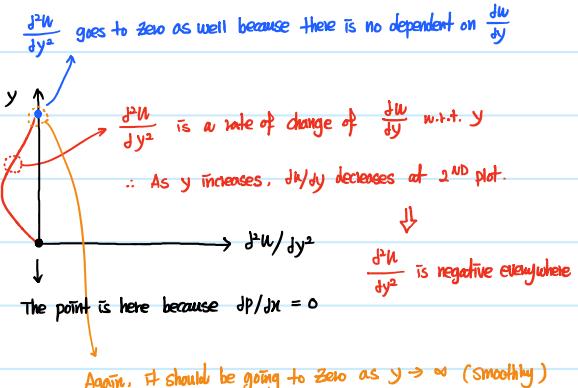
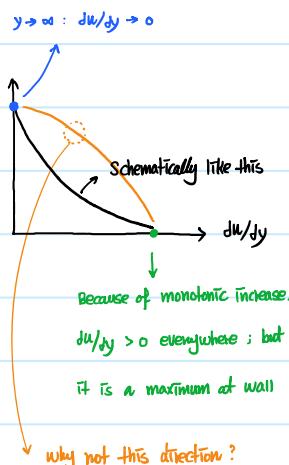
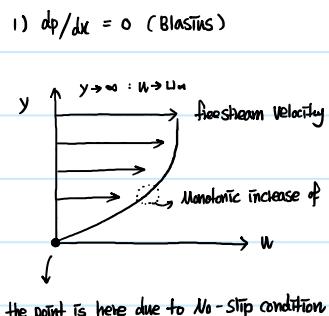
- Assumptions : Laminar, steady, Negligible body force, constant density, and two-dimensional

$$\begin{aligned}
\rho \left( \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) &= \rho \vec{F} - \nabla P + \mu \nabla^2 \vec{u} + \frac{2}{3} \mu \nabla \Delta \\
&\approx 0 \quad (\because \text{steady}) \quad \approx 0 \quad (\because \text{neglect body force}) \quad \approx 0 \quad (\because \rho = \text{const.}) \\
\Rightarrow \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} \quad \text{for } x\text{-direction}
\end{aligned}$$

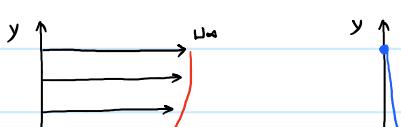
- 4) At the wall :  $u=0$  and  $v=0$  due to no-slip and impermeability

$$\therefore \frac{\partial P}{\partial x} = \mu \frac{\partial^2 u}{\partial x^2}$$

Let's draw different scenarios to explain the effect of pressure gradient on velocity profile. (Graphical reasoning)



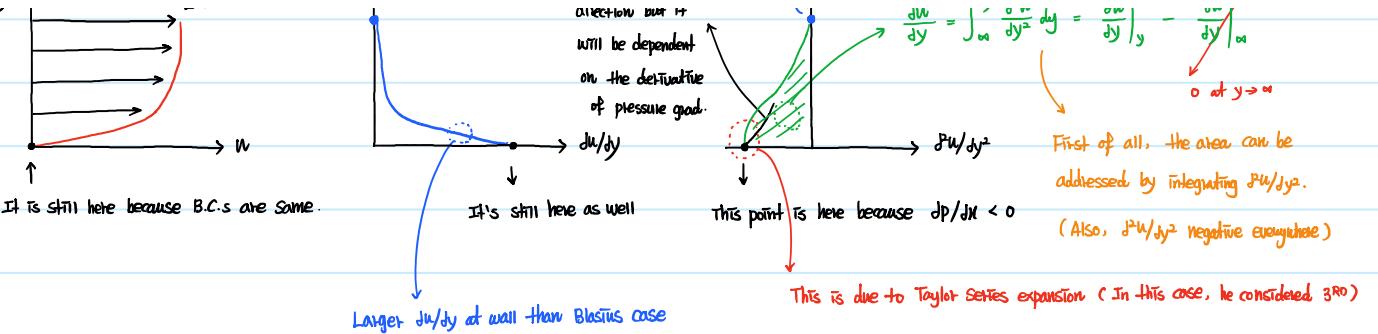
- 2)  $\frac{\partial P}{\partial x} < 0$  (Favorable Pressure gradient) : Accelerated BL.



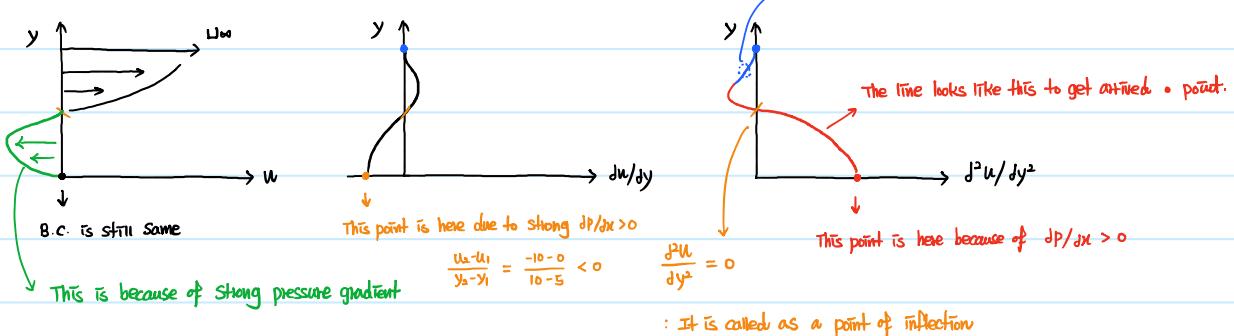
It could be in this direction but it will be dependent on the derivative

This point is still here because of the same reason.

$$\begin{aligned}
\frac{du}{dy} &= \int_{\infty}^y \frac{\partial^2 u}{\partial x^2} dy = \left. \frac{\partial u}{\partial x} \right|_y - \left. \frac{\partial u}{\partial x} \right|_{\infty}
\end{aligned}$$



### 3) $\frac{dp}{dx} > 0$ (Strong) : Separation



- In conclusion, curvature of velocity profile is important indicator.

- Also, it would be good to know
  - Transition (Laminar  $\rightarrow$  Turbulent)

- Separation
  - Laminar
  - Turbulent

- Note that it is not always happen to have separation after transition. Separation could be happen earlier than transition.