Constraint analysis

Saturday, August 26, 2017 19:41

For the glory of God

Master Equation

- Using the master equation derived, we will take a journey with a variety of cases.
- Mattingly's master equation for constraint analysis is as following ;

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{n\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{n\beta W_{TO}}{qS}\right) + C_{D_0} + \frac{R}{qS}\right] + \frac{1}{V}\frac{d}{dt}\left(h + \frac{V^2}{2g}\right)\right\}$$

Case 1 : Constant altitude and speed (Cruise)

States the assumptions

- Constant altitude : $dh/dt = 0$
- Constant speed : $dV/dt = 0$
- Cruise { Clean configuration : $R = 0$; Level flight : n (load factor) $= 1$

In Aeronautics, the load factor is defined as the ratio of the lift to an aircraft to its weight.

$n \equiv \frac{L}{W} \Leftrightarrow L = nW$

(It's technically dimensionless ; however, it sometimes refer to the acceleration of gravity, e.g. $2n \simeq 2g$)

Write the Master Equation

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{\overset{1}{\cancel{n}}\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{\overset{1}{\cancel{n}}\beta W_{TO}}{qS}\right) + C_{D_0} + \underbrace{\cancel{\frac{R}{qS}}}_{0}\right] + \frac{1}{V}\underbrace{\cancel{\frac{d}{dt}}}_{0}\left(h + \underbrace{\cancel{\frac{V^2}{2g}}}_{0}\right)\right\}$$

Hence, we have

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{\beta W_{TO}}{qS}\right) + C_{D_0}\right]\right\}$$

$$\Leftrightarrow \frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left(K_1\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + K_2 + \frac{C_{D_0}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right)$$

Thrust loading = Linear + constant + inverse

- Linear: Related to Induced drag
- constant: Related to interference drag
- inverse: Related to Parasite drag

Let $y = \frac{T_{SL}}{W_{TO}}$ and $x = \frac{W_{TO}}{S}$, then it looks like

$$y = C_1 x + C_2 + \frac{C_3}{x}$$

This implies Thrust required.

In order to find the point at which the $\frac{T_{SL}}{W_{TO}}$ is minimum, take the partial derivatives of $\frac{T_{SL}}{W_{TO}}$ w.r.t. $\frac{W_{TO}}{S}$

$$\frac{dy}{dx} = 0 \Leftrightarrow \frac{d}{d\left(\frac{W_{TO}}{S}\right)}\left(\frac{T_{SL}}{W_{TO}}\right) = 0$$

$$\Leftrightarrow K_1\frac{\beta^2}{\alpha q} + \frac{\cancel{\beta}}{\alpha}\left(-\frac{C_{D_0}}{\cancel{\beta}/q}\frac{1}{\left(\frac{W_{TO}}{S}\right)^2}\right) = 0$$

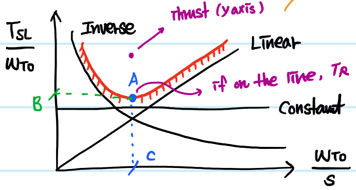
$$\Leftrightarrow K_1\frac{\beta^2}{\cancel{\alpha} q} = \frac{qC_{D_0}}{\cancel{\alpha}}\left(\frac{1}{\left(\frac{W_{TO}}{S}\right)^2}\right)$$

$$\Leftrightarrow \frac{K_1\beta^2}{q^2 C_{D_0}} = \left(\frac{1}{\left(\frac{W_{TO}}{S}\right)}\right)^2$$

$$\Leftrightarrow \frac{\beta}{q}\sqrt{\frac{K_1}{C_{D_0}}} = \frac{S}{W_{TO}}$$

$$\therefore \frac{W_{TO}}{S} = \frac{q}{\beta}\sqrt{\frac{C_{D_0}}{K_1}}$$; wing loading for minimum $\frac{T_{SL}}{W_{TO}}$

Draw the Constraint plot



The plot was roughly generated because we don't know

Note that ; we don't know which term is dominant at this point. → That's why we need to know those coefficients

– : Result plot

• A : Design point

B : minimum $\frac{T_{SL}}{W_{TO}}$

C : wing loading for the minimum

Case 2 : Constant Speed Climb (R/C = const)

States the assumptions

- Constant Speed : $dV/dt = 0$
- Climb { Let's say aircraft already retracted flaps, namely clean configuration : $R = 0$
  It is assumed that lift approximately equals weight : $n \simeq 1$

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{\overset{1}{n}\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{\overset{1}{n}\beta W_{TO}}{qS}\right) + C_{D_0} + \underbrace{\cancel{\frac{R}{qS}}}_{0}\right] + \frac{1}{V}\frac{d}{dt}\left(h + \underbrace{\cancel{\frac{V^2}{2g}}}_{0}\right)\right\}$$

Write the Master Equation

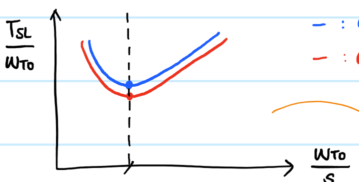
$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{\beta W_{TO}}{qS}\right) + C_{D_0}\right] + \boxed{\frac{1}{V}\frac{dh}{dt}}\right\}$$

By comparing with case 1, the only difference is either this term exists or not.

However, the wing loading for minimum thrust loading would be same because the term is not a function of wing loading. Thus, it will be considered as a constant and disappeared as taken the derivative.

↳ However, minimum thrust loading and the result graph would be different.

Draw the Constraint plot

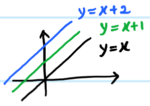


Same wing loading for different minimum thrust loading

– : Climb

– : Cruise

cf. This is why ;



cf. How would your curve shift if power loading is used instead of thrust loading?

∴ The line would generally shift upward as multiplication of V_∞ at all terms.

Case 3 : Horizontal acceleration

States the assumptions

- Level flight { Lift approximately equals to weight : $n \simeq 1$
  Not on the ground → Clean configuration : $R = 0$
  Horizontal flight : $dh/dt = 0$

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{\overset{1}{n}\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{\overset{1}{n}\beta W_{TO}}{qS}\right) + C_{D_0} + \cancel{\frac{R}{qS}}\right] + \frac{1}{V}\frac{d}{dt}\left(\cancel{h} + \frac{V^2}{2g}\right)\right\}$$

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{ \frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{\overset{\to 1}{n}\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{\overset{\to 1}{n}\beta W_{TO}}{qS}\right) + C_{D_0} + \underbrace{\cancel{\frac{R}{qS}}}_{0}\right] + \frac{1}{V}\frac{d}{dt}\left(\underbrace{\cancel{h}}_{0} + \frac{V^2}{2g}\right)\right\}$$

Write the master equation

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{ \frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{\beta W_{TO}}{qS}\right) + C_{D_0}\right] + \frac{1}{V}\frac{d}{dt}\left(\frac{V^2}{2g}\right)\right\}$$

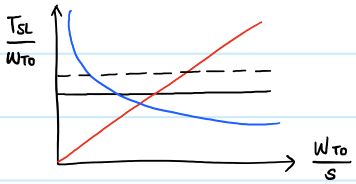
$$\frac{1}{V}\frac{d}{dt}\left(\frac{V^2}{2g}\right) = \frac{1}{2gV}\frac{d}{dt}(V^2)$$

$$= \frac{2V}{2gV}\frac{dV}{dt}$$

$$= \frac{1}{g}\frac{dV}{dt}$$

As can be seen, an additional constant is present.

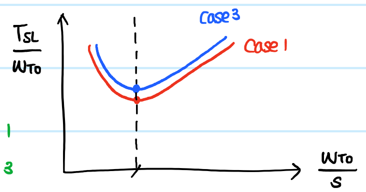
Draw the constraint plot



- Constant
- Linear
- Inverse

Straight line : Case 1

Dashed line : Case 3



Note that we're not able to guarantee that case 2 = case 3 but it will depend on the constant

Case 4 : Constant altitude and speed turn

States the assumptions

- Constant altitude : $dh/dt = 0$
- Constant speed : $dV/dt = 0$
- Not on the ground, namely clean configuration : $R = 0$

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{ \frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{n\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{n\beta W_{TO}}{qS}\right) + C_{D_0} + \underbrace{\cancel{\frac{R}{qS}}}_{0}\right] + \frac{1}{V}\frac{d}{dt}\left(\underbrace{\cancel{h}}_{0} + \underbrace{\cancel{\frac{V^2}{2g}}}_{0}\right)\right\}$$

Write the master equation

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{ \frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{n\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{n\beta W_{TO}}{qS}\right) + C_{D_0}\right]\right\}$$

$$\Leftrightarrow \frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{ K_1 n^2 \frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + K_2 n + \frac{C_{D_0}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\}$$

In order to find the point at which the $\frac{T_{SL}}{W_{TO}}$ is minimum, take the partial derivatives of $\frac{T_{SL}}{W_{TO}}$ w.r.t. $\frac{W_{TO}}{S}$

$$\frac{dy}{dx} = 0 \Leftrightarrow \frac{d}{d\left(\frac{W_{TO}}{S}\right)}\left(\frac{T_{SL}}{W_{TO}}\right) = 0$$

$$\Leftrightarrow \frac{K_1 n^2 \beta^2}{\alpha q} + \frac{\cancel{\beta}}{\alpha}\left(-\frac{qC_{D_0}}{\cancel{\beta}}\frac{1}{\left(\frac{W_{TO}}{S}\right)^2}\right) = 0$$

$$\Leftrightarrow \frac{K_1 n^2 \beta^2}{\cancel{\alpha} q} = \frac{qC_{D_0}}{\cancel{\alpha}}\left(\frac{1}{\left(\frac{W_{TO}}{S}\right)}\right)^2$$

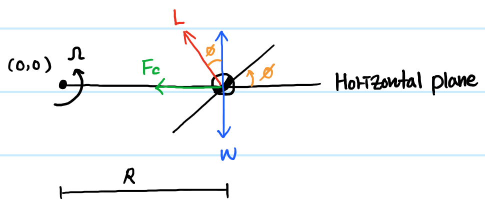
$$\Leftrightarrow \frac{K_1 n^2 \beta^2}{q^2 C_{D_0}} = \left(\frac{1}{\left(\frac{W_{TO}}{S}\right)} \right)^2$$

$$\Leftrightarrow \frac{W_{TO}}{S} = \frac{q}{n\beta}\sqrt{\frac{C_{D_0}}{K_1}}$$; wing loading for minimum $\frac{T_{SL}}{W_{TO}}$ in case 4

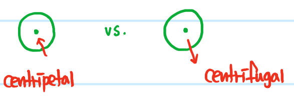
How would you determine the load factor ?

: In order to answer the question, we need a Free Body Diagram (FBD)

· FBD for Case 4 (constant altitude / speed turn)



Ω = Turn rate
ϕ = Bank angle
W = weight
L = Lift ($L = nW$)
F_c = Centripetal force (center seeking)

cf. 

The load factor can be estimated if R and V are given ;

From Pythagoras's theorem, $L^2 = F_c^2 + (-W)^2$

$$\Leftrightarrow L^2 = F_c^2 + W^2$$

$$\Leftrightarrow \frac{L^2}{W^2} = \frac{F_c^2}{W^2} + 1$$

$$\Leftrightarrow n^2 = \frac{F_c^2}{W^2} + 1$$

$$\Leftrightarrow n = \sqrt{\frac{F_c^2}{W^2} + 1}$$

we neglect $\frac{dm}{dt}$ term, $F = \frac{d}{dt}(m\vec{v})$

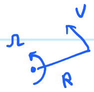
$$= m\frac{d\vec{v}}{dt} + \vec{v}\cancel{\frac{dm}{dt}} \quad \simeq 0$$

$$= m\vec{a}$$

$$= \sqrt{\left(\frac{ma}{mg}\right)^2 + 1}$$

$$= \sqrt{\left(\frac{a}{g}\right)^2 + 1}$$

$$= \sqrt{\left(\frac{V^2}{gR}\right)^2 + 1}$$



; $V = \Omega R \Leftrightarrow \Omega = \frac{V}{R}$

$a = \Omega^2 R$

$= \left(\frac{V}{R}\right)^2 R = \frac{V^2}{R}$

The load factor can be also calculated if the bank angle is given ;

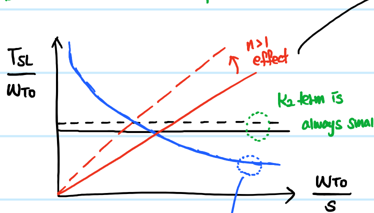
$$n = \frac{L}{W} = \frac{L}{L\cos\phi} = \frac{1}{\cos\phi}$$

$\phi = 0$ rad : $n = 1$

$\phi = 90$ rad : $n \to \infty$ (not good)

cf. 9g is limit # for pilots

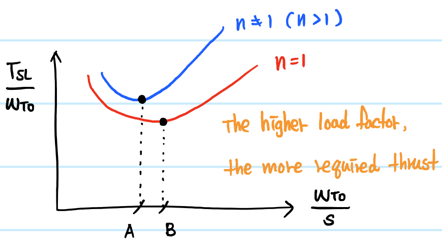
Draw the constraint plot



- Constant
- Linear
- Inverse

Straight line : $n=1$

Dashed line : $n \neq 1$



The higher load factor, the more required thrust

Since we don't see n term in inverse, it should be same as $n=1$ case

Here, A : $\frac{W_{TO}}{S} = \frac{q}{n\beta}\sqrt{\frac{C_{D_0}}{K_1}}$

B : $\frac{W_{TO}}{S} = \frac{q}{\beta}\sqrt{\frac{C_{D_0}}{K_1}}$

Case 5 : Takeoff ground roll with lots of thrust

States the assumptions

- Ground roll $\begin{cases} \text{Ground} : dh/dt = 0 \\ \text{Not clean configuration} : R \neq 0 \end{cases}$ → However, in this case, $R=0$ because of lots of thrust
- Lots of thrust : Neglect all drag terms ($T_{SL} >>> D+R$)

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{ \frac{qS}{\beta W_{TO}}\left[K_1\left(\cancel{\frac{n\beta W_{TO}}{qS}}\right)^2 + K_2\left(\cancel{\frac{n\beta W_{TO}}{qS}}\right) + \cancel{C_{D_0}} + \cancel{\frac{R}{qS}} \right] + \frac{1}{V}\frac{d}{dt}\left(\cancel{h} + \frac{V^2}{2g}\right)\right\}$$

$$\underset{0}{} \quad \underset{0}{} \quad \underset{0}{} \quad \underset{0}{} \quad \underset{0}{}$$

Write the Master Equation

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\frac{1}{V}\frac{1}{2g}\frac{d}{dt}(V^2)$$

$$= \frac{\beta}{\alpha g}\frac{dV}{dt}$$

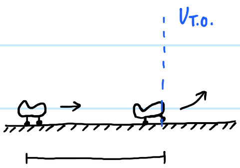
Note that we would prefer to choose a distance as a variable for this case because the length is the most important aspect in takeoff ground roll.

Since $V = \frac{ds}{dt} \Leftrightarrow dt = \frac{ds}{V}$, we can rearrange the equation as following ;

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha g}\frac{dV}{ds/V} \quad \Leftrightarrow \quad ds = \frac{\beta}{\alpha g}\left(\frac{W_{TO}}{T_{SL}}\right) V\,dV$$

By integrating, we have

$$\int_0^{T.O.} ds = \int_0^{V_{T.O.}} \frac{\beta}{\alpha g}\left(\frac{W_{TO}}{T_{SL}}\right) V\,dV$$



S_G : Ground roll distance

$$\Leftrightarrow \quad S_G = \frac{\beta}{2\alpha g}\left(\frac{W_{TO}}{T_{SL}}\right) V_{T.O.}^2$$

; where $V_{T.O.} = K_{T.O.} V_{stall}$ (In general, $K_{T.O.} \simeq 1.2 \sim 1.3$ for safety purpose)

At the ground, $L = W$
$= \beta W_{TO}$

When we think about V_{stall}, $L = \beta W_{TO} = \frac{1}{2}\rho V_{stall}^2 C_{L\max} S$

$$\therefore V_{stall}^2 = \frac{2\beta W_{TO}}{\rho C_{L\max} S}$$

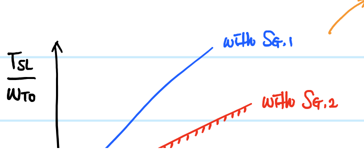
Finally, we have

$$S_G = \frac{\beta}{\alpha g}\left(\frac{W_{TO}}{T_{SL}}\right) K_{T.O.}^2 \frac{2\beta W_{TO}}{\rho C_{L\max} S}$$

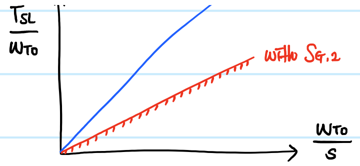
$$\Leftrightarrow \quad \frac{T_{SL}}{W_{TO}} = \frac{\beta^2}{\alpha}\frac{K_{T.O.}^2}{S_G \rho g C_{L\max}}\frac{W_{TO}}{S}$$

Draw the constraint plot

This is a sort of TOGR constraint plot



Here, $S_{G,2} > S_{G,1}$



Here, $S_{G,2} > S_{G,1}$

Case 6 : Service ceiling

Background

- The altitude at which max. $R/C = 0$ is defined as absolute ceiling. (No more left in R/C)
- The altitude at which max. R/C = 100 ft/min is defined as service ceiling. (R/C is still left)
- Both describe how high I can go.
  - e.g. B737 service ceiling : 41,000 ft
- Service ceiling is preferred to use because it represents a practical upper limit of altitude of aircraft.
- Then, how many altitudes does aircraft can go upside?
  - : It is about 10 km because it's hard to obtain air for lift and thrust above the limitation.

States assumption

- No acceleration : $dV/dt = 0$
- $L \cong W$: $n = 1$
- Clean configuration : $R = 0$

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{\overset{1}{\cancel{n}}\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{\overset{1}{\cancel{n}}\beta W_{TO}}{qS}\right) + C_{D_0} + \underset{0}{\cancel{\frac{R}{qS}}}\right] + \frac{1}{V}\frac{d}{dt}\left(h + \underset{0}{\cancel{\frac{V^2}{2g}}}\right)\right\}$$

Write the Master Equation

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{\beta W_{TO}}{qS}\right) + C_{D_0}\right] + \frac{1}{V}\frac{dh}{dt}\right\}$$

Note that this form of the energy equation is not different from that used for the constant speed climb. (Case 2)

Case 7 : Take-off ground roll with not so much thrust

States assumptions

- The R term can't be ignored for this case : $R \neq 0$
  - → The R is defined as following ;

R = Usual drag component + Friction coefficient (weight − Lift)

$\Leftrightarrow \; R = qC_{D_R}S + \mu_{TO}\,(\beta W_{TO} - qC_LS)$

↳ It implies : The faster, the more lift.

∴ Reduce the 2^{ND} term

- On the ground : $dh/dt = 0$

Write the Master Equation

- The starting point for this case is the equation for which the drag terms haven't yet been decomposed into their components ;

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\left(\frac{D+R}{\beta W_{TO}}\right) + \frac{1}{V}\frac{d}{dt}\left(\overset{0}{\cancel{h}} + \frac{V^2}{2g}\right)\right\}$$

$$\Leftrightarrow \frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left(\frac{D+R}{\beta W_{TO}} + \frac{1}{g}\frac{dV}{dt}\right)$$

$$\Leftrightarrow \frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qC_{D_0}S + qC_{D_T}S + qC_{D_R}S + \mu_{TO}\beta W_{TO} - \mu_{TO} q C_L S}{\beta W_{TO}} + \frac{1}{g}\frac{dV}{dt}\right\}$$

$$= \frac{\beta}{\alpha}\left\{\xi_{TO}\frac{q}{\beta}\left(\frac{S}{W_{TO}}\right) + \mu_{TO} + \frac{1}{g}\frac{dV}{dt}\right\} \quad ; \text{ where } \xi_{TO} = C_{D_0} + C_{D_T} + C_{D_R} - \mu_{TO} C_L$$

- Here, $V = \frac{ds}{dt} \Leftrightarrow dt = \frac{ds}{V}$

- Hence,

$$\frac{T_{SL}}{W_{TO}} = \frac{\cancel{\beta}}{\alpha}\xi_{TO}\frac{q}{\cancel{\beta}}\frac{S}{W_{TO}} + \frac{\beta}{\alpha}\mu_{TO} + \frac{\beta}{\alpha g}\frac{VdV}{ds}$$

$$\Leftrightarrow \frac{T_{SL}}{W_{TO}} - \frac{\xi_{TO}q}{\alpha}\frac{S}{W_{TO}} - \frac{\beta}{\alpha}\mu_{TO} = \frac{\beta}{\alpha g}\frac{VdV}{ds}$$

$$\Leftrightarrow \frac{\alpha g}{\beta}\left(\frac{T_{SL}}{W_{TO}} - \frac{\xi_{TO}q}{\alpha}\frac{S}{W_{TO}} - \frac{\beta}{\alpha}\mu_{TO}\right) = \frac{1}{ds}VdV$$

$$\therefore ds = \frac{1}{\frac{\alpha g}{\beta}\left(\frac{T_{SL}}{W_{TO}} - \frac{\xi_{TO}q}{\alpha}\frac{S}{W_{TO}} - \frac{\beta}{\alpha}\mu_{TO}\right)} VdV \quad ; \text{ where } q = \frac{1}{2}\rho V^2$$

By integrating, we have

$$S_G = -\frac{\beta(W_{TO}/S)}{\rho g \xi_{TO}} \ln\left\{1 - \frac{\xi_{TO}}{\left(\frac{\alpha}{\beta}\left(\frac{T_{SL}}{W_{TO}}\right) - \mu_{TO}\right)\frac{C_{L,max}}{k_{TO}^2}}\right\}$$

Draw the constraint plot

- Note that I didn't derive it but brought the result from FWD class material.

Case 8 : Braking roll

- The case 8 will be following the same procedures of case 7 except ;
  - $\beta \simeq 1$ (Takeoff) vs. $\beta \simeq 0.45$ (Landing)
  - Accelerate (Takeoff) vs. De-accelerate (Landing)
  - V_{TO} (Takeoff) vs. $V_{approach}$ (Landing)

Draw a constraint diagram with $V_{approach}$ constraint

$$L = \beta W_{TO} = \frac{1}{2}\rho V_{app}^2 S C_{L,max}$$

$$\Leftrightarrow V_{app} = \sqrt{\frac{2\beta W_{TO}}{\rho S C_{L,max}}}$$

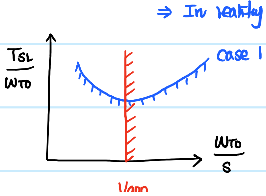
Known : β, ρ, k

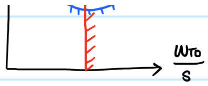
In terms of V_{app}, why is the left feasible ?

$$V_{app} = \sqrt{\frac{2nW}{\rho_\infty S C_{L,max}}}$$

$\Rightarrow$ In reality, $V_{landing} < V_{app}$, why ?

{ Consider the brake ability
Be careful to the structure
(e.g. Landing gear)





(e.g. Landing gear)

Known : β, ρ, K

from safety regulation : V_{app}, $C_{L,max}$

$\Rightarrow$ Hence, we're able to estimate $\frac{W_{TO}}{S}$ = const.

Case 9 : Cruise condition with the assumption that we add internal weapons

States assumptions

- It should be the same as case 1 because this flight is in cruise.

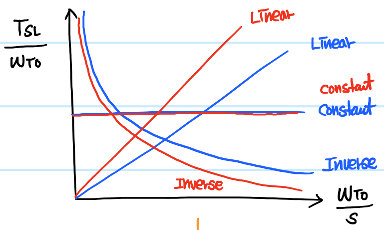
Write the Master Equation

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left(K_1 \frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + K_2 + \frac{C_{D_0}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right)$$

Draw a constraint plot

Note that adding internal weapons implies to increase in wing loading.

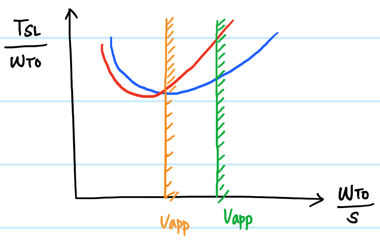
→ ∴ Linear term ↑ , inverse term ↓ , Same constant term



– : Cruise w/o weapons

– : Cruise with weapons

Then, we have



Let's say we have Vapp constraints ;

– Vapp with weapons

– Vapp w/o weapons

$$\uparrow V_{app} = \sqrt{\frac{2\beta W_{TO}}{\rho S C_{L,max}}} \uparrow$$

Case 10 : Cruise condition with the assumption that we add external weapons, e.g. Bomb

States assumptions

- It should be the same as case 1 because this flight is in cruise.

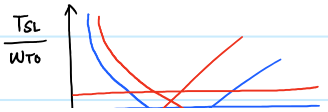
Write the Master Equation

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left(K_1 \frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + K_2 + \frac{C_{D_0}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right)$$

Draw a constraint plot

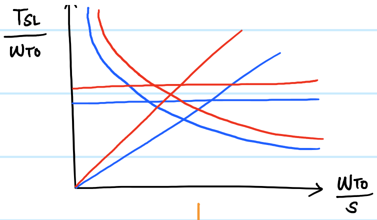
Note that adding external weapons implies to increase in weight which turns out drag increase

→ All drag terms increase ∴ Linear, Constant, inverse ↑



– : Cruise without External weapons

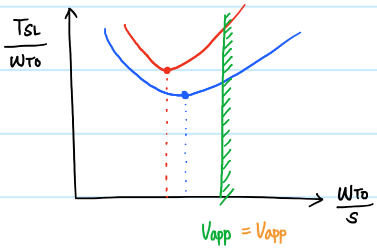
– : Cruise with External weapons



— : Cruise without External weapons

— : Cruise with External weapons

Then we have,



Here, Vapp term wasn't affected by drag increase

(assume that you dropped off the bomb before landing)

Other Cases

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{n\beta W_{TO}}{qS}\right)^2 + K_2\left(\frac{n\beta W_{TO}}{qS}\right) + C_{D_0} + \frac{R}{qS}\right] + \frac{1}{V}\frac{d}{dt}\left(h + \frac{V^2}{2g}\right)\right\}$$

what happens if the airplane cruise at higher altitude ?

- Less air density ⇔ Less Drag, Low q terms

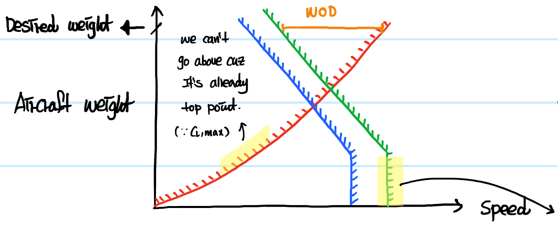
what happens if the airplane cruise at higher Mach number ?

- High q terms

what if it is in Mars ?

- Less density
- Less gravity ⇔ higher load factor ($n = \frac{L}{mg}$)

Carrier - Launch wind over deck



— : Airspeed required

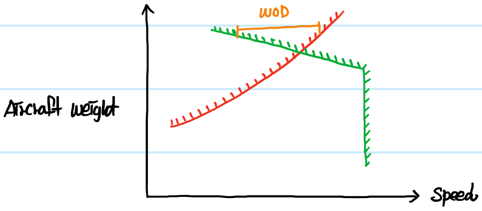
— : Cat deadload Capability

— : Cat plus A/C thrust

— : Wind over deck (WOD)

Constraint would be chopped outside because of the speed limit

Recovery wind over deck (Recovery headwind)



— : Approaching gear performance

— : A/C touchdown speed

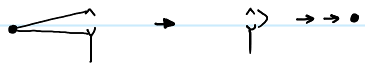
— : wind over deck (WOD)

observations,,

- The airspeed required (Red-line) is coming from the Lift coefficient Equation.

$$L = \frac{1}{2}\rho_\infty U_\infty^2 S C_L \Leftrightarrow W = \frac{1}{2}\rho_\infty U_\infty^2 S C_{L,max} \therefore W \propto U_\infty^2$$

· Cat deadload capability means ;



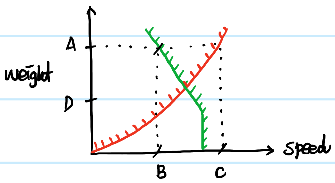
; Just by the device

· Cat plus A/C thrust means : the device + A/C own thrust ∴ Thus, more speed

· The desired weight could be achieved with the speed matched in the curve line if we have a long runway.

↳ However, the problem is that we're dealing with the carrier operations.

(Even with Cat plus A/C thrust, there is a constraint)

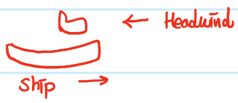


- For A point, C speed enables to achieve it
- However, due to the green constraint, for A, we will have B speed.
- Unfortunately, with B speed, we'll be not able to obtain enough lift to the weight.
- Hence, our weight would be decreased to somewhere around D.
- This will cause another problem because carrier aircraft prefer to carry out lots of missiles.

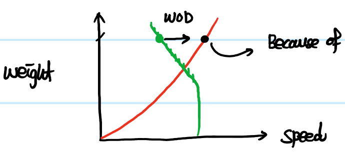
In order to solve this issue, wind over deck is used (or introduced)

For takeoff, to get more speed, the ship will be getting faster, so that

aircraft lift would be advantaged.



Also, the aircraft can have more speed.. so that



Because of the speed added, (∵ WOD)

the desired weight is achieved