Pressure coefficient

Sunday, September 17, 2017 19:17

For the glory of God

Pressure coefficient (C_p)

- Pressure, by itself, is a dimensional quantity.

  ↳ However, a dimensionless pressure would be used in Aerodynamics because of the usefulness of dimensionless parameter.

- Such a quantity is the pressure coefficient (C_p).

- C_p can be defined as following and it is used throughout Aerodynamics from incompressible to hypersonic flow.

$$C_p \equiv \frac{P - P_\infty}{q_\infty} \quad ; \text{ where } q_\infty = \frac{1}{2}\rho_\infty U_\infty^2$$

cf. Be careful to put ∞ in case that we choose other local points

Here, P represents static pressure at the point at which C_p is being evaluated.

(It is not necessary being at the wall)

- For incompressible flow, C_p can be expressed in terms of velocity only ;

  - According to the Bernoulli's Equation, we have

$$P_\infty + \frac{1}{2}\rho_\infty U_\infty^2 = P + \frac{1}{2}\rho U^2$$

$$\Leftrightarrow \quad P - P_\infty = \frac{1}{2}\rho\left(U_\infty^2 - U^2\right) \quad ; \text{ Here, } \rho_\infty = \rho = \text{const for incompressible flow}$$

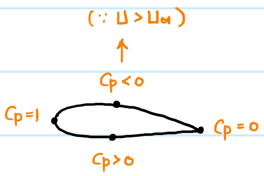
  - By plugging it into the definition of C_p, we have

$$C_p = \frac{\frac{1}{2}\rho\left(U_\infty^2 - U^2\right)}{\frac{1}{2}\rho_\infty U_\infty^2} = 1 - \left(\frac{U}{U_\infty}\right)^2 \quad ; \quad C_p = f(\text{velocity}) \text{ only}$$

Here, note that this holds for incompressible flow only.

  - There are two main results ;

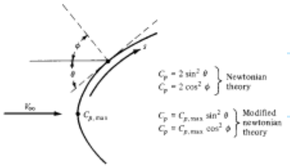
$$\begin{cases} \text{At stagnation point, } U = 0 \rightarrow C_p = 1 \\ \text{If } U = U_\infty, \ C_p = 0 \end{cases}$$



- For compressible flows,

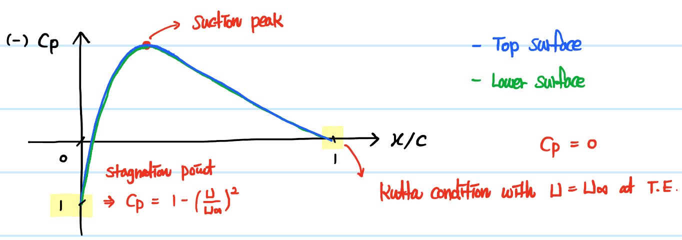
  - C_p at a stagnation point is greater than 1.0

    ↳ Based on Newtonian theory, $C_p = 2\sin^2\theta = 2\cos^2\phi$

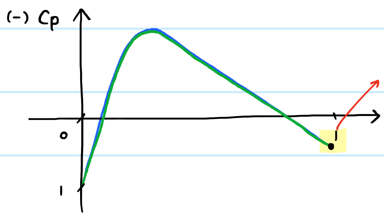


C_p plot

a) Inviscid / Incompressible / $\alpha = 0$ / Symmetric Airfoil



b) Viscous / Incompressible / $\alpha = 0$ / Symmetric



$C_p \approx 1 - \left(\frac{U}{U_\infty}\right)^2 > 0$

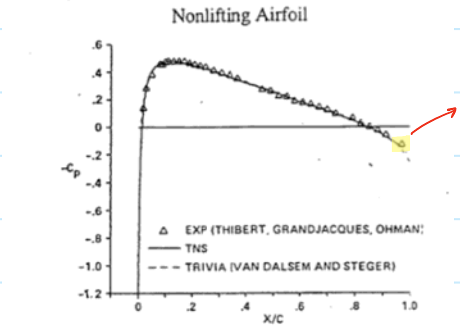
Kutta condition with $\boxed{U < U_\infty}$ at T.E.

This is due to the viscosity.

Strictly speaking, this is not valid because Bernoulli's eq doesn't hold for viscous phenomena.

↳ But we got same result because $C_p = \frac{P - P_\infty}{q_\infty}$ ($\because V\downarrow$, $P\uparrow$)

c) Viscous / Compressible / $\alpha = 0$ / Symmetric

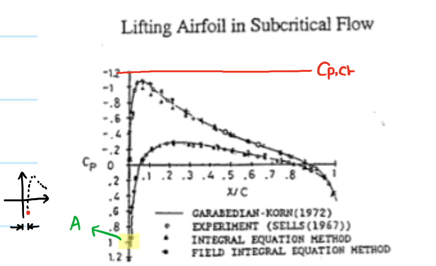


Pressure coefficient comparisons (NACA 0012 airfoil section, $\Lambda = 0$ deg, large aspect ratio, TR = 1.0, $M_\infty = 0.5$, $\alpha = 0$ deg, $Re = 3 \times 10^6$).

It means they are attaching at T.E.

(Kutta condition)

d) Viscous / Compressible / $\alpha = 2°$ / Symmetric / Subcritical flow

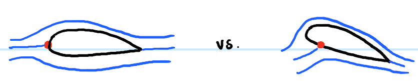


Subcritical pressure distribution on NACA 0012 airfoil; $M_\infty = 0.63$, $\alpha = 2$ deg, 40 surface elements.

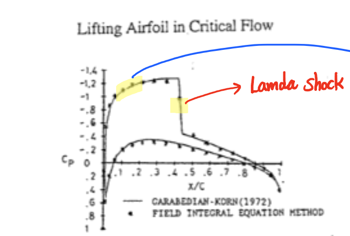
In terms of A,

∴ A stagnation point is not still in the leading edge

when α is increased such as ; • : stagnation point



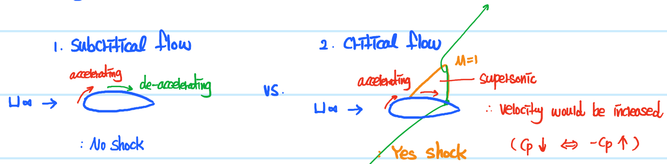
e) Viscous / Compressible / $\alpha = 2°$ / Symmetric / Critical flow



Supercritical pressure distribution on NACA 0012 airfoil; $M_\infty = 0.75$, $\alpha = 2$ deg, 40 surface elements.

why are they increasing instead of decreasing ?

very strong shock



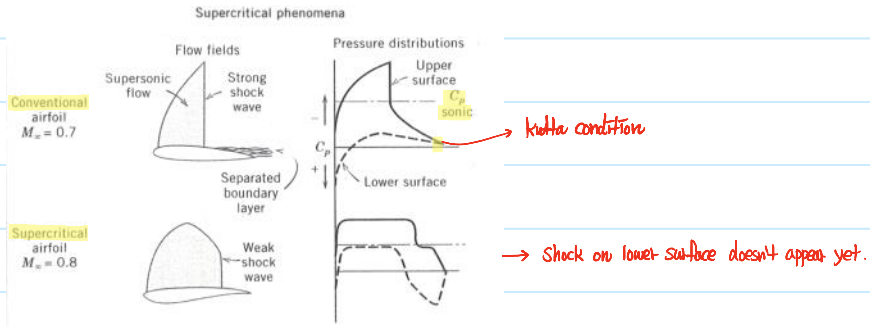
∴ velocity would be increased

($C_p \downarrow \Leftrightarrow -C_p \uparrow$)

After this point,

pressure would be dramatically increased

d) Supercritical phenomena (viscous / compressible - transonic)



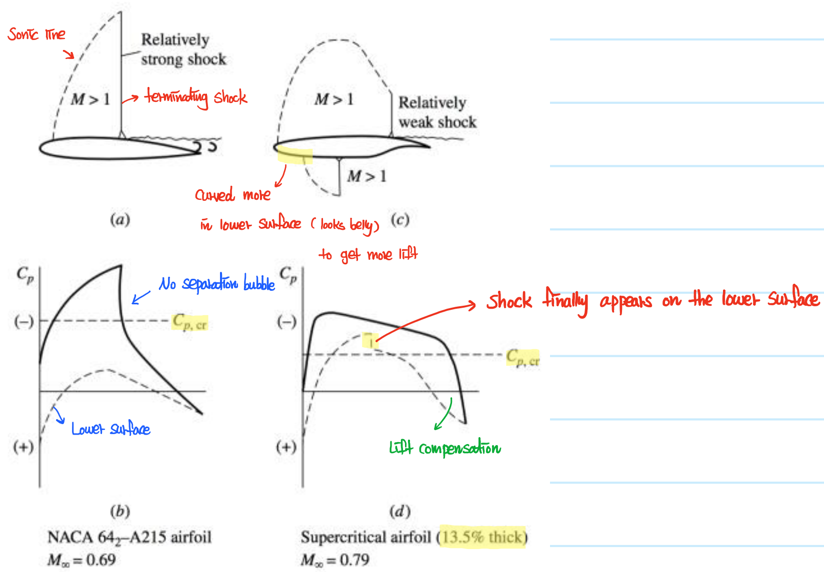


Figure 11.19 Standard NACA 64-series airfoil compared with a supercritical airfoil at cruise lift conditions. *(From Reference 32.)*

e) Viscous / Compressible - Supersonic / $\alpha = 2°$ / Bi-convex airfoil

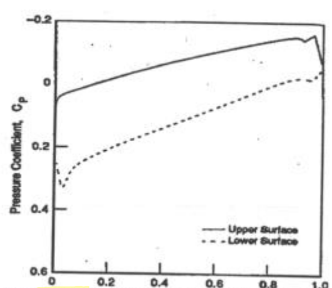
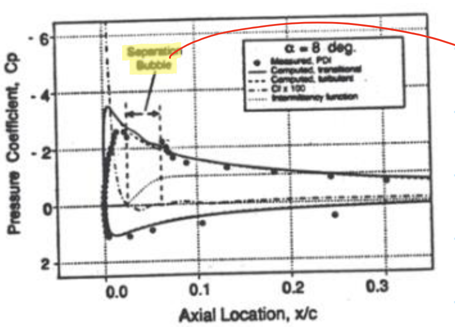
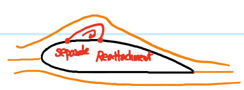
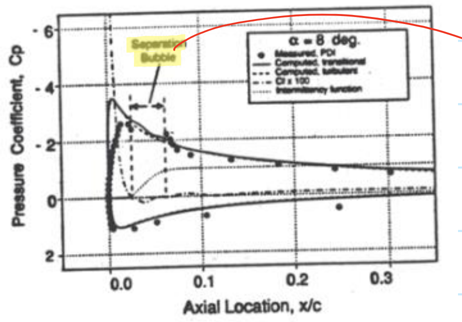


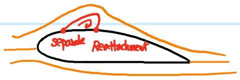
Figure 4. Lifting airfoil in supersonic flow. This is the pressure distribution was found using the FLO67 Euler code. The wing consisted of biconvex airfoils of 5% thickness with a chord ranging from 22.5 ft at the centerline to 5.45 ft at the tip. The pressure distribution was taken at 50% semispan. The calculations used a Mach of 2.0 and an angle of attack of 4 deg. This example came from the Journal of Aircraft Vol. 28, No. 11, Nov 1991, pg. 722

f) Separation bubble

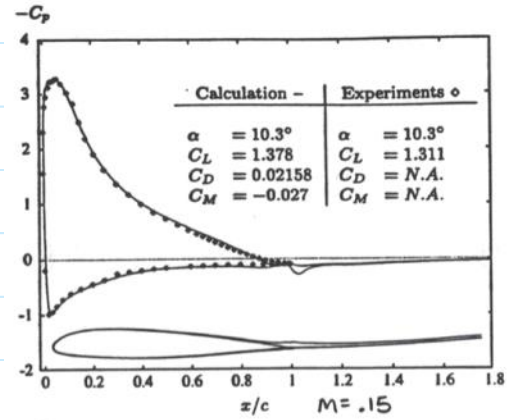


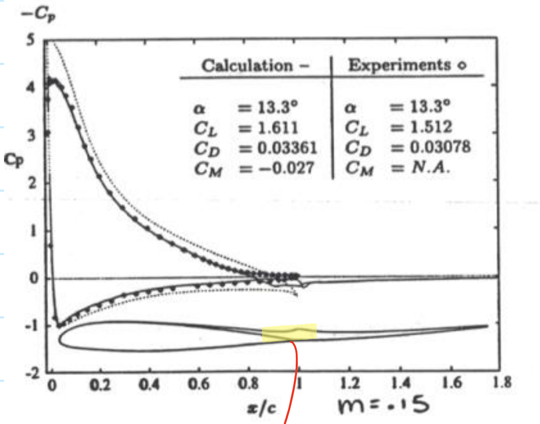






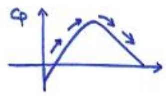
g) Limitation of CFD





The reason why ... see the below

usually, we would say that streamline shows us where fluid Elements are going. Let's discuss about it in detail.



Based on the geometry of the airfoil, needless to say, Cp would be decreased smoothly. However, in CFD, there should be a separation at Trailing edge because of the limit. The separation will be greater and greater as α increases.

$\alpha = 0$ vs.

Let's imagine that you are on the fluid Element (■) in CFD. You might feel that geometry, which is actually streamline, is slightly changed ( ) at trailing edge. So, you probably have experienced a little bit increase or flat Cp at the point. This is the reason why Cp distributions are so complex at T.E. as CFD is analyzed. (with high Angle of attack)