Transonic Aerodynamics

Tuesday, July 18, 2017 20:04

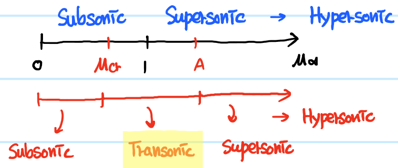
For the glory of God

§. Introduction

· Transonic flow is defined as a flow where subsonic and supersonic flows are existed (or mixed) simultaneously.

→ It is characterized by the presence of shocks on the body.

↳ Two situations can exist { predominantly subsonic + supersonic / subsonic + predominantly supersonic



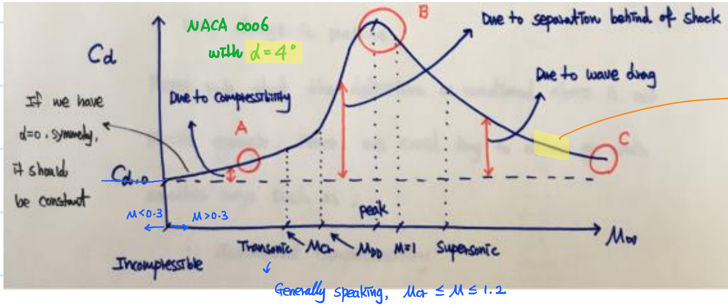
Blue : Perspective of the flow

Red : Perspective of the body

{ Lower bound : M_{cr} (critical Mach number)
Upper bound : A point (where the shock moves all the way to T.E.)

§. Drag coefficient vs. Mach number

· If the flow is passing over an airfoil, what does C_d vs. M look like?



This region has obviously more drag than subsonic because of the wave drag.

· A is caused by compressibility effects. However, in most cases, it's very small, therefore ≃ constant

$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}}$; P-G rule $\tau \equiv -\frac{1}{v}\frac{dv}{dp}$ cf. $\tau_{gas} \gg \tau_{water}$

∴ C_p is always greater than $C_{p,0}$ ↳ Change in volume of the fluid element per unit change in pressure

· At B point, it seems that the peak point of C_d is very before $M=1$. This is because ;

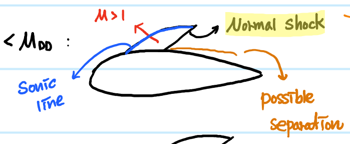
: Take a look 5)

1) $M_\infty < M_{cr}$:  Small drag (Mainly due to viscous)

2) $M_\infty = M_{cr}$: 

A is a point where $M=1$

→ At some freestream Mach number, the local flow becomes sonic at a single point on the upper surface where the flow reaches its highest speed locally.

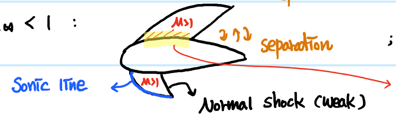
3) $M_{cr} < M_\infty < M_{DD}$: 

; Drag increase slightly

why does the shock generate?

⇒ In order to recover the back pressure up to P_∞.

(why not at the beginning point?)

4) $M_{DD} < M_\infty < 1$: 

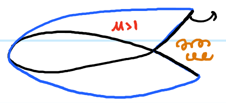
; Large drag increase

⇒ even though $M=1$, shock depends on the back pressure as well

→ As the freestream Mach number increases further, a region of supersonic flow develops, the shock moves after and becomes stronger. Also, a supersonic region and shock

→ As the freestream Mach number increases further, a region of supersonic flow develops, the shock moves after and becomes stronger. Also, a supersonic region and shock wave develops on the lower surface as well.

5) $M_\infty \le 1$:



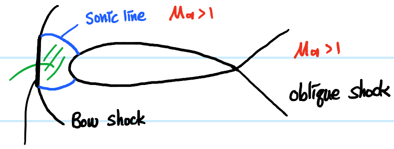
(related to wave drag) (highest drag)

; Peak point of C_d

one of the reasons why it is located in peak point

→ As the Mach number approaches one, the shock moves all the way to the T.E.

6) $M_\infty \ge 1$:

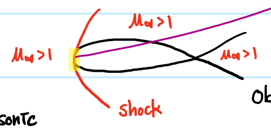


; Drag decrease

a kind of normal shock so that it is expected to see subsonic region.

→ When $M_\infty \ge 1$, a bow shock appears just ahead of the airfoil and the shock at T.E. becomes oblique shock.

7) $M_\infty > 1$:



(It becomes weaker than 6)

There will always be a subsonic bubble on the L.E. (Bow shock)

↔ We can neglect it if it is small enough.

(e.g. BL → subsonic → thin → neglect)

From now on, not transonic

· At C region, Mach number independence (High Mach number Inviscid flow)

: At high Mach numbers, certain aerodynamic quantities become independent of M.

§. Why is transonic flow important?

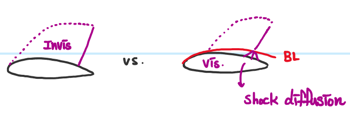
· Commercial transport aircraft prefer to operate in transonic flow regime because this allows the range of the aircraft to be maximum.

· The flow over the tip regions of helicopter rotors and high speed propellers in high speed forward flight often become transonic.

· Turbomachinery components such as compressors and turbines are often designed to operate in the transonic flow regime.

· Fighters perform at high subsonic or transonic speeds.

cf. Inviscid vs. Viscous shock



§. Governing Equation

cf. For more details, please see TSD note.

Navier-Stokes Equation → Euler Equation → Transonic potential Equation → Transonic small disturbance Equation (TSD)

Inviscid

Irrotational / Steady

Isentropic

If the shock waves are weak, buffeting is not present; so that the flow is assumed steady

Actually, while isentropic is convenient, transonic flows are never isentropic. Nevertheless, isentropic flow approximations are acceptable when weak shock waves alone are present.

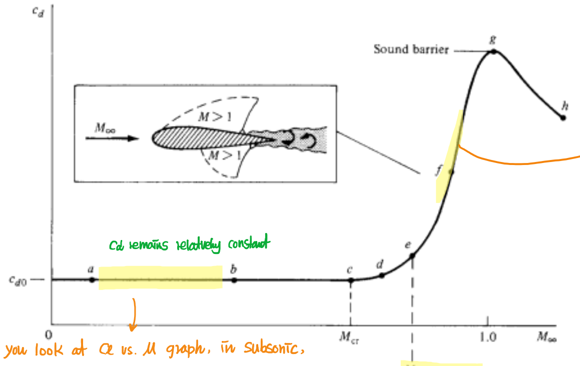
Nevertheless, isentropic flow approximations and ...

waves alone are present.

It usually includes one or more shock.

§. Drag - Divergence Mach number (M_{D-D})

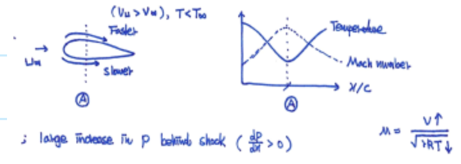
· M_{D-D} is the Mach number at which the Aerodynamic drag begins to increase rapidly as the Mach number continues to increase.



Beyond M_{D-D}, Cd can become very large,

typically, increasing by a factor of 10

cf. Mach number & Temperature



; large increase in p behind shock ($\frac{dp}{dx} > 0$)

$M = \frac{V\uparrow}{\sqrt{\gamma R T}\downarrow}$

If you look at C_L vs. M graph, in subsonic,

- C_L is proportional to the Mach number.

However, Cd is not. why? Cd isn't important to pressure at subsonic.

we encounter a point where Cd suddenly starts to increase.

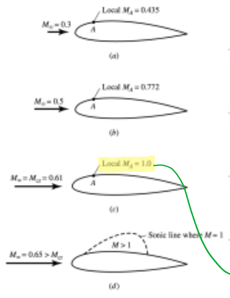
↳ In fact, there are two types of people who have defined the M_{D-D}.

a) Boeing and Lockheed Martin : They focus on the point at which M_{D-D} is rapidly increasing.

b) other company in US : They focus on the slope at which the slope changes suddenly.

§. Critical Mach number (M_{cr})

· M_{cr} is the Mach number at which sonic flow is first achieved on the airfoil surface.



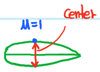
Here,

- Let point A represent the location on the airfoil surface where the pressure is minimum,

hence, where M is a maximum.

- As M_∞ increases, M_A also increases.

→ Sonic condition will be achieved at maximum thickness. For symmetric airfoil,



· One of the most important problems in high-speed Aerodynamics is the determination of M_{cr} of a given airfoil ;

→ This is because at values of M_∞ slightly above M_{cr}, the airfoil experiences a dramatic increase in drag coefficient.

∴ It is desirable to have M_{cr} as high as possible.

· Then, how would we estimate the M_{cr} ?

- Let p_∞ and p_A represent the static pressures in the freestream and at point A, respectively.

- For isentropic flow, where the total pressure P_0 is constant. (ρ_0 and T_0 as well)

From the isentropic relation, we have $\frac{P_0}{P} = \left[1 + \frac{1}{2}(\gamma-1)M^2\right]^{\frac{\gamma}{\gamma-1}}$

- Since P_0 = const, P_0 should be same at two location A and B.

$$P_{0_A} = P_{0_B} \iff P_A\left[1+\frac{1}{2}(\gamma-1)M_A^2\right]^{\frac{\gamma}{\gamma-1}} = P_B\left[1+\frac{1}{2}(\gamma-1)M_B^2\right]^{\frac{\gamma}{\gamma-1}}$$

- Let A be freestream ($M_A = M_\infty$) and B be where $M=1$ ($M_B = 1$)

$$\frac{P_B}{P_A} = \frac{P}{P_\infty} = \left[\frac{1+\frac{1}{2}(\gamma-1)M_\infty^2}{1+\frac{1}{2}(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}}$$

why am I doing this?

: For compressible, q_∞ is no longer an accurate measure.

$$C_p \equiv \frac{P-P_\infty}{q_\infty}$$

; where $q_\infty = \frac{1}{2}\rho_\infty U_\infty^2 = \frac{1}{2}\frac{\gamma P_\infty}{\gamma P_\infty}\rho_\infty U_\infty^2 = \frac{\gamma}{2}P_\infty\left(\frac{\rho_\infty}{\gamma P_\infty}\right)U_\infty^2$

$a^2 = \frac{\gamma P}{\rho}$

$= \frac{\gamma}{2}P_\infty\frac{U_\infty^2}{a_\infty^2}$

$= \frac{\gamma}{2}P_\infty M_\infty^2$

- From the definition of Cp expressed in terms of M_∞, we have

$$C_p = \frac{2}{\gamma M_\infty^2}\left(\frac{P}{P_\infty}-1\right) \iff C_{p_{cr}} = \frac{2}{\gamma M_{cr}}\left[\left(\frac{1+\frac{1}{2}(\gamma-1)M_{cr}}{1+\frac{1}{2}(\gamma-1)}\right)^{\frac{\gamma}{\gamma-1}}-1\right]$$

↳ $C_{p_{cr}}$ is a unique function of M_{cr}

- From Prandtl-Glauert compressibility correction, we have

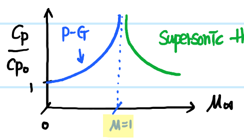
$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}} \iff C_{p,cr} = \frac{C_{p,0}}{\sqrt{1-M_{cr}^2}}$$

It is based on the linearized perturbation velocity potential equation. (∴ Limited to thin airfoil at small α)

cf. Especially, the linearized theory is not valid for transonic region. (Around $M \simeq 1$)

⇒ This is because sonic shock waves are instantaneous (and thus very non-linear) changes in the flow.

↳ It violates the assumption.



Supersonic thin airfoil theory, $C_p \propto \frac{1}{\sqrt{M_\infty^2-1}}$

This is so-called P-G singularity

- Hence, we have

$$\frac{C_{p,0}}{\sqrt{1-M_{cr}^2}} = \frac{2}{\gamma M_{cr}}\left[\left(\frac{1+\frac{1}{2}(\gamma-1)M_{cr}}{1+\frac{1}{2}(\gamma-1)}\right)^{\frac{\gamma}{\gamma-1}}-1\right]$$

; Mathematical approach (Solve it with respect to M_{cr})

step 1) obtain the low-speed incompressible value of the pressure coefficient by either experimental or theoretical.

step 2) Using the compressibility correction equation, plot the variation of Cp with M_∞.

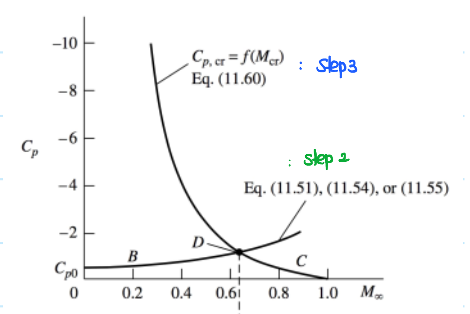
step 3) Using the Cp equation from linearized theory, plot the variation of Cp with M_∞.

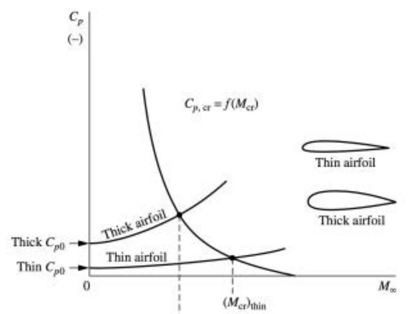
step 4) Find the intersection of two plots, which in turn, the critical Mach number

cf. Result table looks like ;

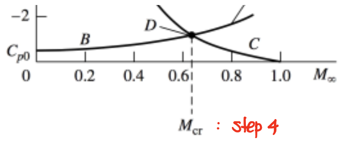
M	P-G	linearized theory
0.1	⋮	⋮
0.2	⋮	⋮
⋮		
0.7	-0.742	-0.743

It would be M_{cr}

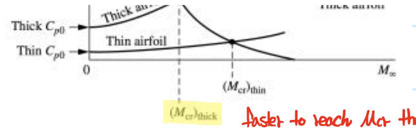




; Graphical approach



: step 4



faster to reach M_{cr} than thin airfoil

· Keep in mind that this is not an exact determination of M_{cr}. However, it is quite useful for preliminary design.

§. Area Rule

· Air Force supersonic capabilities in the early 1950s, such as the Convair F-102 delta-wing airplane, ran into difficulty, namely sound barrier. (in level flight)

↳ The thrust of jet engines at that time simply could not overcome the large peak drag near Mach 1.

· For this reason, Aerodynamicists focused on reducing the large drag rise.

· One of researchers, who is Whitcomb, experimented with different wing body combinations in a transonic wind tunnel.

⇒ Whitcomb reasoned that ;

e.g. Total cross-sectional area (wing area + fuselage area = total)

" The variation of cross-sectional area for an airplane should be smooth, with no discontinuities "

This means that ;

In the region of the wings and tails, the fuselage cross-sectional area should decrease to compensate for the addition of wings and tail cross-sectional area.

· This design philosophy is called the Area rule

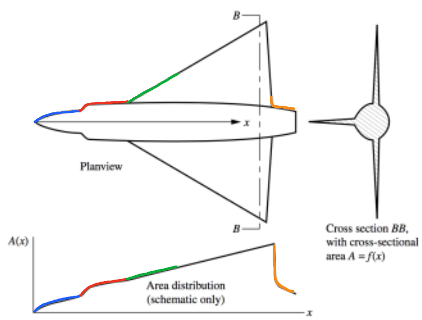


Figure 11.16 A schematic of a non-area-ruled aircraft.

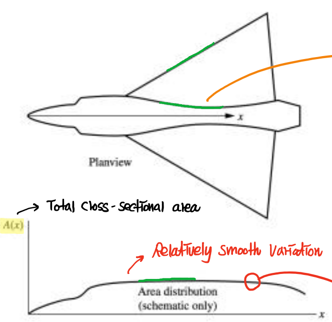


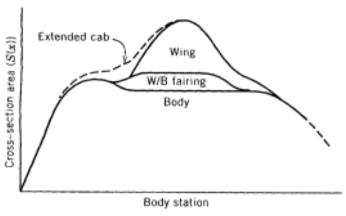
Figure 11.17 A schematic of an area-ruled aircraft.

Sometimes it is called as ;

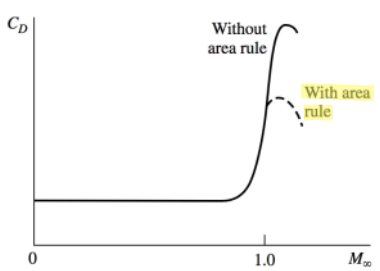
" Coke bottle "

(Design concept for a complete wing)

To be specific,



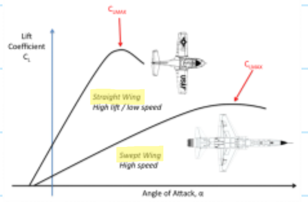
So, it successfully reduced the peak drag near Mach 1.



cf. The area rule would be more proper to fighter than civil aircraft.

↳ This is because civil aircraft prefers to flight below M_{dd} to save fuel.

< Flow separation at tip first >

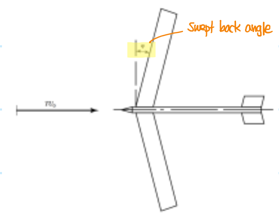


§. Swept - Wing Concept (Delay M_{cr})

· The concept of swept wings for high-speed flight was first introduced at the fifth Volta Conference in Rome in 1935 by Dr. Adolf Busemann.

· Busemann reasoned that ;

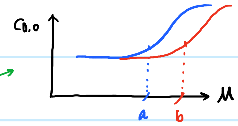
" The flow over a wing is governed mainly by the component of velocity perpendicular to the leading edge. If the wing is swept, this component will decrease and consequently the supersonic wave drag will decrease "



: from Busemann's original paper

Here, Swept back effects are ;

- Delay the drag rise



a : $M_{D\text{-}D}$ for no sweep wing

b : $M_{D\text{-}D}$ for Swept back angle

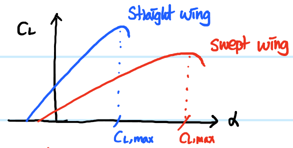
- put center of gravity back, which in turn increase static stability

- Looks good

- Less Bending moment at the root

(∵ less lift than straight wing)

↳ But less drag as well ∴ L/D would be good !

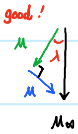


$U_\infty > U_n$

(Flowing over the wing)

· To be more specific,

< No sweep angle > vs. < Swept back angle >

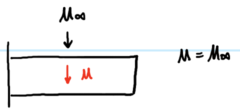


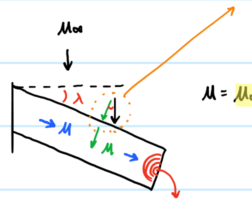
$M = M_{\infty,n} = M_\infty \cos\lambda$; Normal component

$M = M_{\infty,s} = M_\infty \sin\lambda$; Spanwise component

Typically, the angle (λ) is designed to satisfy ;

$M_n < M_{cr}$





$M = M_\infty \cos\lambda$; where λ = swept back angle

Flow over the wing is mainly governed by normal component

Mach number over the wing

Flow separation at tip (∵ Spanwise component of flow keeps flowing toward the tip)

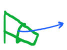
· Therefore, { For no sweep, $M = M_\infty$

For swept back, $M = M_\infty \cos\lambda$ i.e. $M = \frac{\sqrt{3}}{2} M_\infty$ for $\lambda = 30°$

↳ Hence, less lift though.

In order to avoid it and to improve the efficiency,

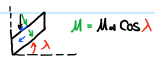
{ wingtip device  it also reduce C_{D_i}

wing fence  prevent the air from flowing sideways

· what about swept-forward design ?

: It also makes the $M_{D\text{-}D}$ delay, however, it is typically not used in civil aircrafts because of stability issues.

On the other hand, military aircraft pursue the design because of high maneuverability at high α. cf. Swept forward



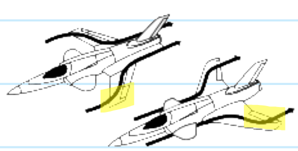
§ The Supercritical Airfoil (Delay $M_{D\text{-}D}$)

· It is well known that high critical Mach number is very desirable, indeed necessary, for high-speed subsonic aircraft.

· We know that thinner airfoils have higher values of M_{cr} ; however, the airfoil requires

{ certain thickness for structural strength

There must be a room for the storage of fuel



· This prompts the following question ; for an airfoil of given thickness,

" How can we delay the large drag rise to higher Mach numbers ? "

: For forward swept wing, ailerons remained unstalled at high α because the air flows inward which is heading to the root.

(For swept back, it flows outward)

→ This leads to the design of a new family of airfoils so-called Supercritical airfoils.

The purpose of a supercritical airfoil is to increase the value of $M_{D\text{-}D}$ although M_{cr} may change very little.

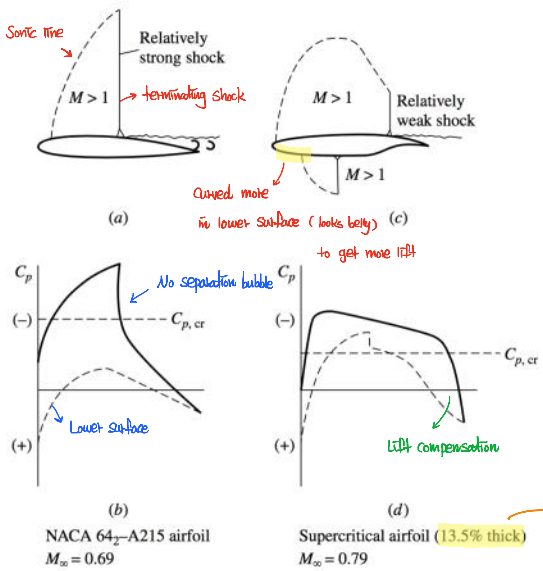


Figure 11.19 Standard NACA 64-series airfoil compared with a supercritical airfoil at cruise lift conditions. *(From Reference 32.)*

This is actually verified by experiment.

Here,

- The supercritical airfoil has a relatively flat top
  - : Thus, encouraging lower local values of M
    - ↳ In turn, the terminating shock is weaker, thus less drag.

Since it is flat, the forward 60% of the airfoil has negative camber.

- To compensate, the lift is increased by having extreme positive camber on the reward 30%. → This is why cusplike shape of bottom near T.E.

NASA conducted a research and announced that ;

they chose a certain thickness best for the lift characteristics.

What is positive and negative camber?

: See the below

· Therefore,

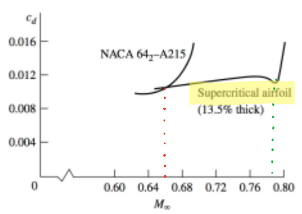
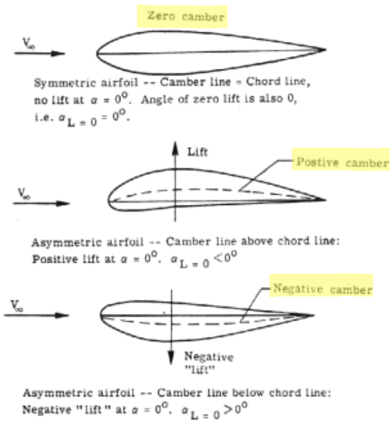


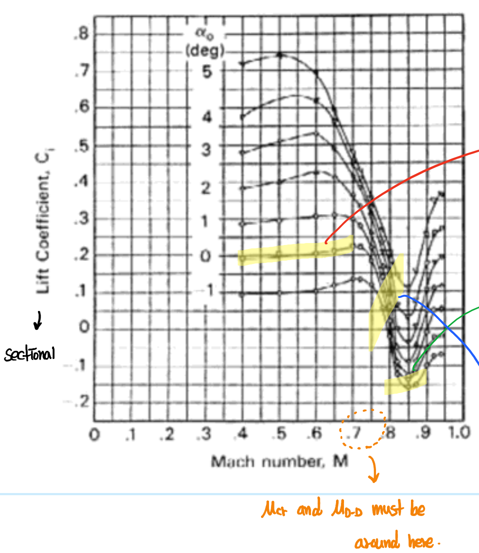
Figure 11.20 The drag-divergence properties of a standard NACA 64-series airfoil and a supercritical airfoil.

cf. Many variation of the supercritical airfoil are now used by the aircraft industry on modern high-speed airplane design such as B757 and B767.



§. Lift coefficient vs. Mach number

· By similar way, we will see the transonic effect on C_ℓ vs. M



Based on P-G correction,

$$C_\ell = \frac{C_{\ell,0}}{\sqrt{1-M_\infty^2}}$$

After this point,

Shock appears on lower surface as well

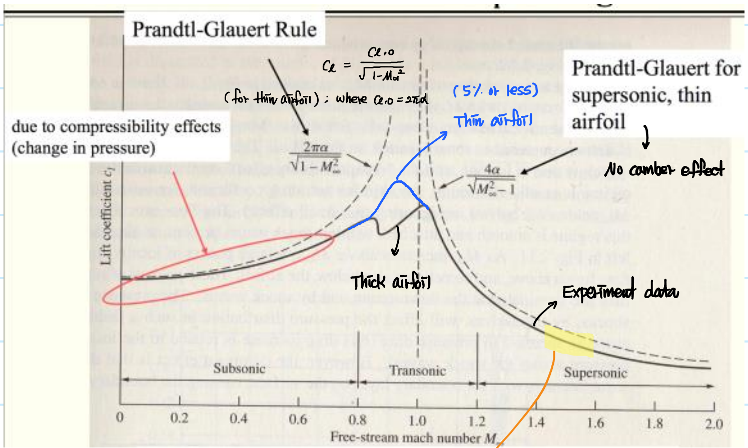
Dramatically drops because of the shock on the top.

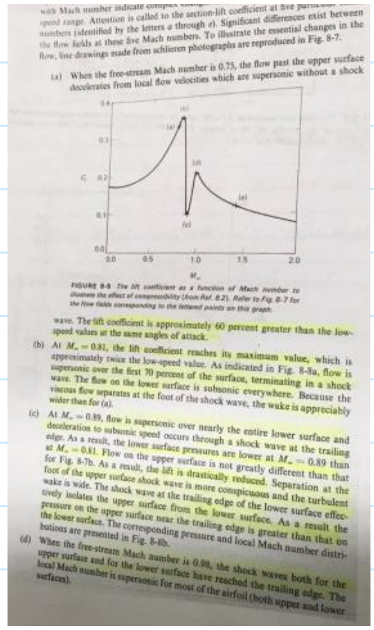
Prandtl-Glauert Rule

TRANSONIC FLOW PAST UNSWEPT AIRFOILS

The lift coefficient measurements presented in Ref. 8.2 as a function of Mach number are reproduced in Fig. 8-6. The data indicate that the flow is essentially unchanged up to approximately one-half the speed of sound. The variations in the lift coefficient with Mach number indicate complex changes in the flow field through the transonic speed range. Attention is called to the section-lift coefficient at five particular Mach numbers (identified by the letters a through e). Significant differences exist between the flow fields at these five Mach numbers. To illustrate the essential changes in the flow, line drawings made from schlieren photographs are reproduced in Fig. 8-7.

(a) When the free-stream Mach number is 0.75, the flow past the upper surface decelerates from local flow velocities which are supersonic without a shock





with Mach number indicate complex ... Attention is called to the section-lift coefficient at five particular ... speed range. ... numbers (identified by the letters a through e). Significant differences exist between the flow fields at these five Mach numbers. To illustrate the essential changes in the flow, line drawings made from schlieren photographs are reproduced in Fig. 8-7.

(a) When the free-stream Mach number is 0.75, the flow past the upper surface decelerates from local flow velocities which are supersonic without a shock

FIGURE 8-8 The lift coefficient as a function of Mach number to illustrate the effect of compressibility (from Ref. 8.2). Refer to Fig. 8-7 for the flow fields corresponding to the lettered points on this graph.

wave. The lift coefficient is approximately 60 percent greater than the low-speed values at the same angles of attack.

(b) At $M_\infty = 0.81$, the lift coefficient reaches its maximum value, which is approximately twice the low-speed value. As indicated in Fig. 8-8a, flow is supersonic over the first 70 percent of the surface, terminating in a shock wave. The flow on the lower surface is subsonic everywhere. Because the viscous flow separates at the foot of the shock wave, the wake is appreciably wider than for (a).

(c) At $M_\infty = 0.89$, flow is supersonic over nearly the entire lower surface and deceleration to subsonic speed occurs through a shock wave at the trailing edge. As a result, the lower surface pressures are lower at $M_\infty = 0.89$ than at $M_\infty = 0.81$. Flow on the upper surface is not greatly different than that for Fig. 8-7b. As a result, the lift is drastically reduced. Separation at the foot of the upper surface shock wave is more conspicuous and the turbulent wake is wide. The shock wave at the trailing edge of the lower surface effectively isolates the upper surface from the lower surface. As a result the pressure on the upper surface near the trailing edge is greater than that on the lower surface. The corresponding pressure and local Mach number distributions are presented in Fig. 8-8b.

(d) When the free-stream Mach number is 0.98, the shock waves both for the upper surface and for the lower surface have reached the trailing edge. The local Mach number is supersonic for most of the airfoil (both upper and lower surfaces).

Why is it lower than subsonic case ?

- First of all, the shock effect

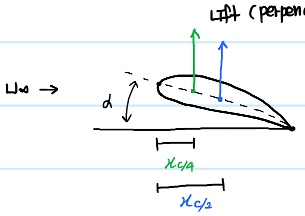
- Second, the easiest way is to compare : $C_\ell = \frac{2\pi\alpha}{\sqrt{1-M_\infty^2}}$ vs. $C_\ell = \frac{4\alpha}{\sqrt{M_\infty^2-1}}$

Let's say $\alpha = 1$, M_∞ $\begin{cases} \text{Subsonic : } 0.1 \\ \text{Supersonic : } 2 \end{cases}$

$\therefore$ C_ℓ (subsonic) $\cong 2\pi$ vs. C_ℓ (Supersonic) $\simeq \frac{4}{\sqrt{3}}$

§. Moment coefficient vs. Mach number

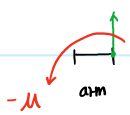
· In order to answer the question, let's assume that we are now talking about cambered airfoil at $\alpha = 4°$.



- $c/2$: Supersonic
- $c/4$: Subsonic

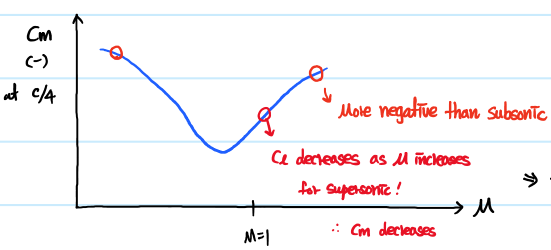
· When we think about the moment, moment = Force × arm

e.g.



; where we have a sign convention ↻ (+)

· Obviously, Supersonic case has more negative moment than subsonic one because of the length of arm.



cf. If symmetry with $\alpha = 0$,

$L = 0 \Leftrightarrow C_m = \text{const}$

⇒ This is the opposite to C_ℓ phenomena.

; Here,

- as Mach increases, lift increases. $\therefore$ moment increases

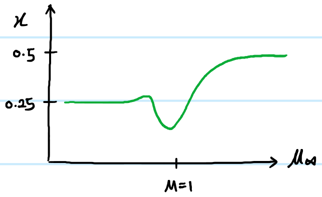
; Here,

- as Mach increases, lift increases. ∴ moment increases

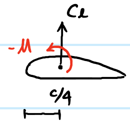
(However, keep in mind that we have (-) sign on the y axis)

This is because ;

§. M_∞ vs. Location of Aerodynamic center

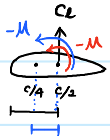


· For subsonic,



: Assume Lift is generated at c/4

· For supersonic,



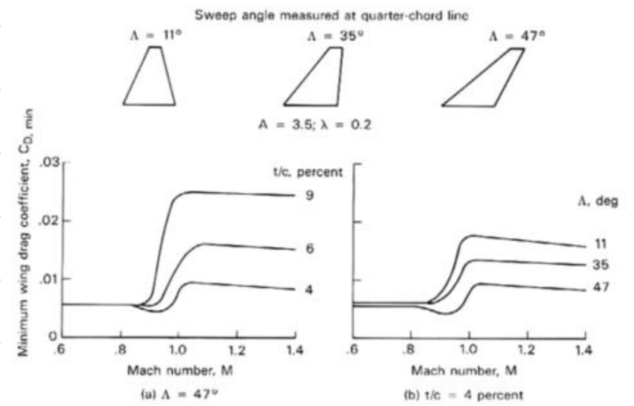
: Assume Lift is generated at c/2

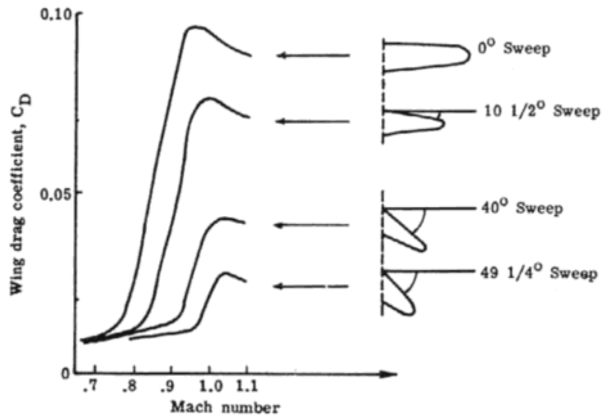
$$C_{m,c/4} = C_{m,c/2} - C_\ell\,(c/2 - c/4) \leftarrow$$

Here, make sure that we are focusing on c/4 point in the plot

§. C_D vs. Mach number - Thickness ratio and swept back angle

· Eventually, more thickness → makes faster to have M_{cr} and M_{DD}





Source: history.nasa.gov