Panel method

Wednesday, September 6, 2017 19:07

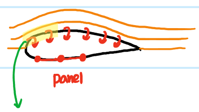
For the glory of God

Introduction

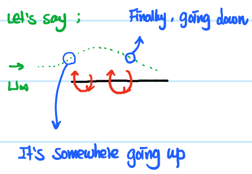
valid only if at small angle of attack

- The advantage of thin airfoil theory is that closed-form expressions are obtained for the Aerodynamic coefficients.
  → The results compare favorably with experimental data for airfoils of about 12% thickness or less.
- In reality, the airfoils on many low-speed airplanes are thicker than 12 percent. Also, we are frequently interested in high angle of attack such as occur during takeoff. Finally, we are sometimes concerned with the generation of Aerodynamic lift on other body shapes such as automobiles.
- For these reasons, the vortex panel method has come into widespread use since the early 1970s.
  (we limit our attention to two-dimensional flow)

Wrapping the vortex sheet over the complete surface of the body



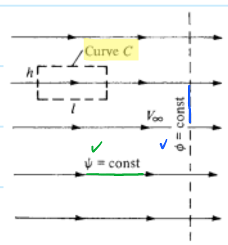
Four different types of Elementary flow (Inviscid / Incompressible / Irrotational)



- In order to understand the panel method, we may need to take a look this section first.

a) Uniform flow

- Uniform flow is a viable elementary flow for use in building more complex flows.
- Consider a uniform flow with velocity U_∞ oriented in the positive x direction. $\begin{cases} u = U_\infty \\ v = 0 \end{cases}$



For the uniform flow,

$$\nabla\cdot\vec{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \therefore \text{Incompressible}$$

$$\nabla\times\vec{u} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = i\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) - j\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) + k(\cdot) = 0$$

$\therefore$ Irrotational

$$\Gamma = \oint_c \vec{u}\cdot d\vec{s} = -U_\infty l - h\cdot 0 + U_\infty l + h\cdot 0 = 0$$

↳ Circulation around any closed curve in the uniform flow is zero.

- Hence, a velocity potential for the uniform flow can be obtained such that $\nabla\phi = \vec{u}$

$$\frac{\partial\phi}{\partial x} = u = U_\infty \qquad \frac{\partial\phi}{\partial y} = v = 0$$

→ Integrating, we have $\phi = U_\infty x + f(y)$, $\phi = g(x) + \text{const}$ → They must be same. $\therefore \phi = U_\infty y + \text{const}$

ϕ is always used to obtain the velocity by $\nabla\phi = \vec{u}$. Since the derivative of constant is zero, we can drop the constant without any loss of rigor. $\therefore \phi = U_\infty y$

- By similar way, It's easier to visualize ψ than ϕ.

$$\frac{\partial\psi}{\partial y} = u = U_\infty \rightarrow \psi = U_\infty y + g(x)$$

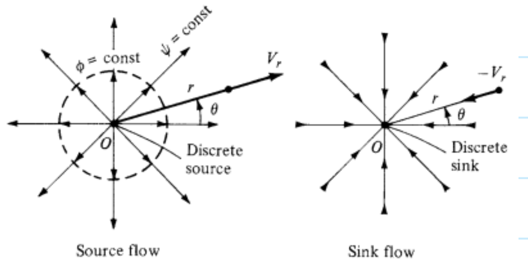
$$-\frac{\partial\psi}{\partial x} = v = 0 \rightarrow \psi = g(y)$$

→ $\therefore \psi = U_\infty y$

Since the equation of a streamline is given by $\psi = \text{const}$, the streamlines for the uniform flow are given by $U_\infty y = \text{const} \Leftrightarrow y = \text{const}$

b) Source / Sink flow : A cylindrical coordinate is preferred.

· Consider a flow where all the streamlines are straight lines emanating from a central point O.



For this flow, $\begin{cases} V_r = \dfrac{c}{r} \\ V_\theta = 0 \end{cases}$; Sink if $c<0$, Source if $c>0$

· It is a physically possible incompressible flow ($\nabla \cdot \vec{u} = 0$) at every point except the origin. → At here, $\nabla \cdot \vec{u}$ becomes ∞

· It is irrotational at every point. ($\nabla \times \vec{u} = 0$)

Continuity equation with constant density ; (2D)

$$\frac{\rho}{r}\frac{\partial}{\partial r}(u_r r) + \frac{\rho}{r}\frac{\partial u_\theta}{\partial \theta} = 0$$

$$\Leftrightarrow \frac{\rho}{r}\left(u_r \frac{\partial r}{\partial r} + r\frac{\partial u_r}{\partial r}\right) = 0 \quad (\because u_\theta = 0)$$

$$\Leftrightarrow \frac{\rho u_r}{r}\frac{\partial r}{\partial r} + \rho\frac{\partial u_r}{\partial r} = 0$$ → At the origin ($r=0$), then infinity

· For ϕ and ψ, for the derivation process, see Aerodynamics binder

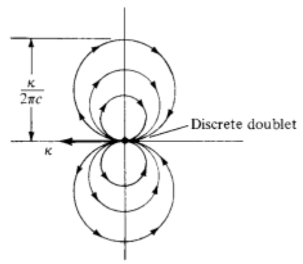
$$\phi = \pm \frac{\lambda}{2\pi}\ln r$$; where $\lambda = 2\pi r V_r$ (It is defined as a strength)

$$\psi = \pm \frac{\lambda}{2\pi}\theta$$

c) Doublet flow

· There is a special, degenerate case of a source - sink pair that leads to a singularity called a doublet.

· For the derivation, please see the binder as well. ↳ Superposition



$$\phi = \frac{\kappa}{2\pi}\frac{\cos\theta}{r}$$; where $\kappa = \ell\lambda$ (strength of the doublet)

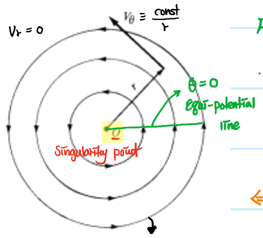
$$\psi = -\frac{\kappa}{2\pi}\frac{\sin\theta}{r}$$

d) Vortex flow ; cf. The reason why the vortex is chosen is to represent something in the phenomena in flow

· Consider a flow where all the streamlines are concentric circles about a given point.

· For the vortex flow, let the velocity along any given circular streamline be constant.

(However, let it vary from one streamline to another inversely with distance from the center)



one of streamlines

For this flow,

· It is easily shown that it's a physically possible incompressible flow at every point

$$\frac{\rho}{r}\frac{\partial}{\partial r}(u_r r) + \frac{\rho}{r}\frac{\partial u_\theta}{\partial \theta} = 0$$; 2D continuity eq. with constant density

$$\Leftrightarrow 0 + \frac{\rho}{r}\frac{\partial u_\theta}{\partial \theta} = 0 \quad (\because u_r = 0)$$

$$\Leftrightarrow \frac{\rho}{r}\frac{\partial}{\partial \theta}\left(\frac{c}{r}\right) = 0$$ ∴ Satisfied!

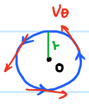
· However, the vortex flow is irrotational ($\nabla \times \vec{u} = 0$) at every point except the origin. ($r=0$)

Vorticity is infinite.

· So, what happens at $r = 0$?

$$\Gamma = \oint_c \vec{u} \cdot d\vec{s} \quad \circlearrowright \oplus$$

$$= -V_\theta \times 2\pi r$$



$d\vec{s} = 2\pi r$ for the streamline

(the circulation around a given circular streamline of radius r)

Γ → strength of the vortex flow (fixed)

(the circulation around a given circular streamline of radius r)

Hence, $\Gamma = -V_\theta 2\pi r \Leftrightarrow V_\theta = -\frac{\Gamma}{2\pi r}$ → strength of the vortex flow (fixed)

(Initially, we defined) $V_\theta = \frac{C}{r}$

$\therefore$ Const $= -\frac{\Gamma}{2\pi}$

Based on the Stoke's theorem, $\oint_c \vec{u} \cdot d\vec{s} = \iint_s (\nabla \times \vec{u}) \, d\vec{s}$ ⊕

Then, $-V_\theta 2\pi r = -\iint_s (\nabla \times \vec{u}) \, d\vec{s}$

$\Leftrightarrow V_\theta 2\pi r = (\nabla \times \vec{u}) \, d\vec{s}$; At the origin, the area inside the small circle will become infinitesimally small

$\Leftrightarrow \frac{V_\theta 2\pi r}{d\vec{s}} = \nabla \times \vec{u}$

→ But in reality, because of viscosity, the singularity isn't sustainable. (Diffuses away from the core)

$\therefore$ as $r \to 0$, $d\vec{s} \to 0$, therefore $\nabla \times \vec{u} \to \infty$ (infinite)

· For velocity potential and stream function for the vortex flow,

equi-potential lines are given by θ = const. (straight radial line from the origin)

$V_r = \frac{\partial \phi}{\partial r} = 0 \rightarrow \phi = \text{const} + f(\theta)$

$V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\Gamma}{2\pi r} \rightarrow \phi = -\frac{\Gamma}{2\pi}\theta + g(r)$

$\therefore \phi = -\frac{\Gamma}{2\pi}\theta$

$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 0 \rightarrow \psi = \text{const} + f(r)$

$V_\theta = -\frac{\partial \psi}{\partial r} = -\frac{\Gamma}{2\pi r} \rightarrow \psi = \frac{\Gamma}{2\pi}\ln r + g(\theta)$

$\therefore \psi = \frac{\Gamma}{2\pi}\ln r$

ψ = const : streamline

It states that the streamlines of vortex flow are given by r = constant (i.e. streamlines are circles)

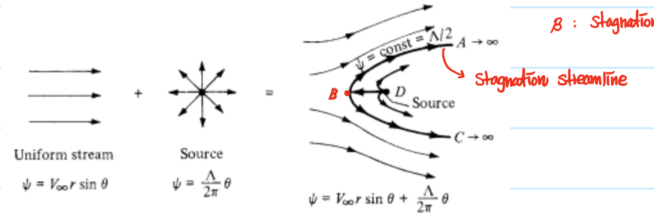
Superimposed the Elementary flows

(Linear Combination)

cf. Under the assumptions, superimpose is possible because Laplace Eq. ($\nabla^2\phi = 0$) is a linear.

a) Uniform + Source

· Consider a polar coordinate system with a source of strength λ located at the origin. And then, superimpose on this a uniform stream with velocity U_∞ moving from left to right



B : Stagnation point ($V_r = V_\theta = 0$)

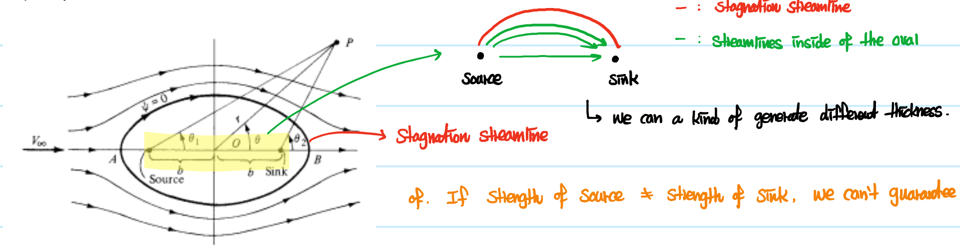
b) Uniform + Source + Sink

· Consider a polar coordinate system with a source and sink placed a distance b to the left and right of the origin.

· Superimpose a uniform flow with velocity U_∞

— : stagnation streamline

— : streamlines inside of the oval



↳ we can a kind of generate different thickness.

cf. If strength of source ≠ strength of sink, we can't guarantee the closed circle.

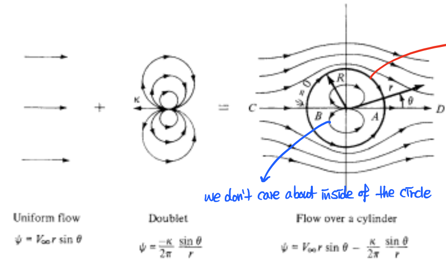
; This is called as Oval shape solved first by Rankine

c) Uniform + Doublet

- Consider the addition of a uniform flow with velocity U_∞ and a doublet of strength κ

↳ The direction of it is upstream, facing into the uniform flow.

- This is referred to non-lifting flow over a circular cylinder, which is one of the most classic problems in Aerodynamics.



Stagnation streamline

($\psi = 0$ ∴ $r = R$)

Here, the radius of the cylinder could be changed by both U_∞ and κ

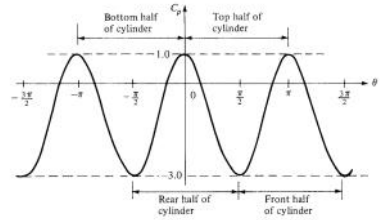


Figure 3.29 Pressure coefficient distribution over the surface of a circular cylinder; theoretical results for inviscid, incompressible flow.

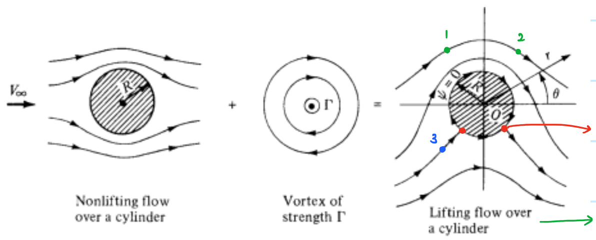
d) Uniform + Doublet + Vortex

- This is referred to lifting flow over a cylinder. cf. For detail in derivation, please see the Aerodynamic binder

- In real life, there are non-symmetric Aerodynamic forces.

•1 = •2 with respect to ↑y axis ⇒ No drag

•1 ≠ •3 w.r.t. → x axis ⇒ Yes Lift



Stagnation point

depends on the strength

Panel method

- At the beginning of this note, we mentioned that thin airfoil theory wouldn't be proper to the reality.
- Also, we have already dealt with the lifting flow over a circular cylinder.

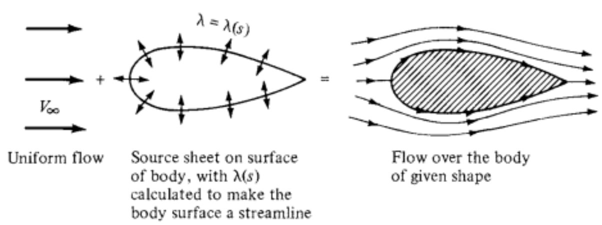
↳ This indirect method of starting with a given combination of elementary flows.

However, seeing what body shape comes out of it can hardly be used in a practical sense for bodies of arbitrary shape.

- For this reason, we consider the panel method which is a numerical method with a high-speed digital computer.

↳ In fact, the numerical solution of potential flows by both source and vortex panel methods has revolutionized in low-speed flows.

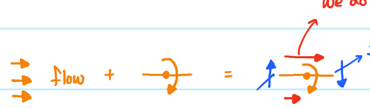
- For instance, superposition of a uniform flow and a source sheet on a body of given shape



; Source panel method (No lift)

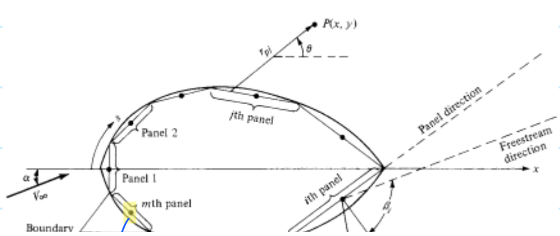
we do see the discontinuity of velocity

It could be canceled out.



- In order to discuss about the lift, we may need to use the Vortex panel method. Let's dive into the method.
- We now return to the original idea of wrapping the vortex sheet over the complete surface of the body.

↳ There exists no closed-form analytical solution for $\gamma(s)$

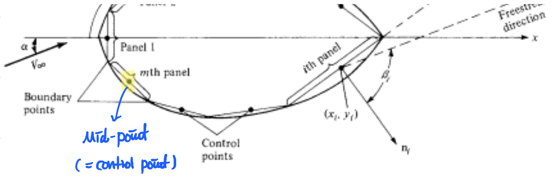


; We wish to find $\gamma(s)$ numerically such that the body surface becomes a streamline of the flow.

↓ using panel method

(The local tangential velocities to the airfoil are equal to the local values of γ)

$\gamma \equiv u_1 - u_2 = u_1 - 0$



Mid-point (= control point)

Surface becomes a streamline of the flow.

↓ using panel method

(The local tangential velocities to the airfoil are equal to the local values of γ)

$\gamma = u_1 - u_2 = u_1 - 0$

Once we have obtained the value $\gamma(s)$, { $\gamma = u_1$ (tangential vel.) ⇒ Bernoulli's Eq. ⇒ Pressure

This is a sort of the vortex panel method. $\Gamma = \sum_{j=1}^{n} \gamma_j S_j$ ⇒ K-J theorem ⇒ Lift

- Let the vortex strength $\gamma(s)$ per unit length be constant over a given panel (But allow it to vary from one panel to the next)
- The mid-point of each panel is a control point at which the boundary condition is applied.
  - ↳ At the control points, the normal component of the velocity is zero such that the body surface becomes a streamline of the flow.

$$V_{\infty,n} + V_n = 0$$

This is the crux of the vortex panel method.

This is termed a first-order method.

For high-order, second-order method assumes a linear variation of γ over a given panel.

- We must also satisfy the Kutta condition.
  - ↳ In order to impose the condition on the solution of the flow, $\gamma_i = -\gamma_{i-1}$ must be included.

where does it come from ?

cf. please note that ;

The length of each panel can be different. Their length and distribution over the body are up to your discretion.

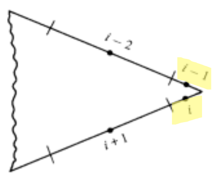


Figure 4.32 Vortex panels at the trailing edge.

→ Kutta condition $\gamma(T.E) = 0$

$\Leftrightarrow \gamma_i + \gamma_{i-1} = 0$

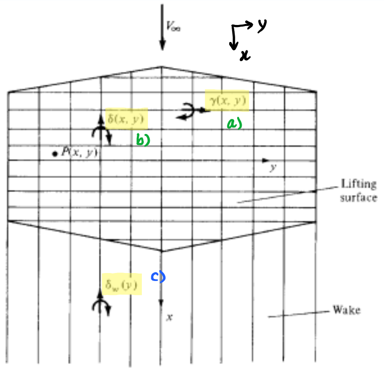
$\therefore \gamma_i = -\gamma_{i-1}$

- These boundary conditions (we discussed) are applied to the control points of all the panels.
  - → we obtain a system of n linear equations with n unknowns.
  - → Solve it numerically to get $\gamma(s)$
  - → using the $\gamma(s)$, we may be able to obtain : Γ, L, u, P
- Although the method may appear to be straightforward, its numerical implementation can sometimes be frustrating.

  e.g. the results are sensitive to the number of panels used.
- The problems are usually overcome in different ways { High-order method / Combination of source and vortex panels

Vortex Lattice Method (Analysis of finite wing)

- Prandtl's lifting line theory gives reasonable results for straight wings at moderate to high aspect ratio.
  - ↳ However, for some cases such as swept wing and Low AR wing, the theory is inappropriate.
- Let's extend the theory by placing a series of lifting lines on the plane of the wing as following ;



The two vortex sheets

a) The one with vortex lines running parallel to y with strength γ

$\gamma = \gamma(x, y)$; Spanwise vortex strength distribution

b) The other with vortex lines running parallel to x with strength δ

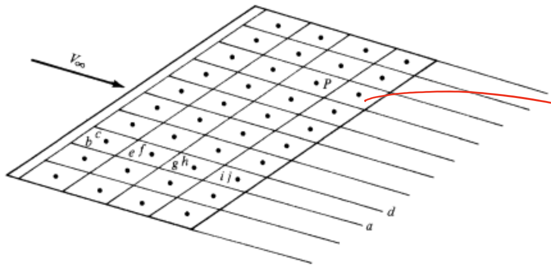
$\delta = \delta(x, y)$; Chordwise vortex strength distribution

Therefore, at any given point on the surface, the strength of the lifting surface is given by both γ and δ.

Note that c) downstream of the trailing edge

: No spanwise vortex lines but only trailing vortices. → Chordwise vortices

· The wing planform is divided into a number of panels as following ;



; We may want the wing planform to be a stream surface of the flow

The entire wing is covered by this lattice of horseshoe vortices

↳ Each of different unknown strength Γ_n

↱ It's said to be Von Neuman Boundary condition. (Impenetrability)

⇒ can't penetrate..

- Control points on the panels can be chosen where the flow tangency condition is applied.

↳ The net normal flow velocity is zero.

- Find $\gamma(x, y)$ and $\delta(x, y)$ such that the sum of the induced $w(x, y)$ and the normal component of the free-stream velocity to be zero for all points on the wing.

- Calculate Γ and Lift

↳ the lifting surface and the wake vortex sheet induce a normal component of velocity.

↳ It can be calculated by Biot-Savart law